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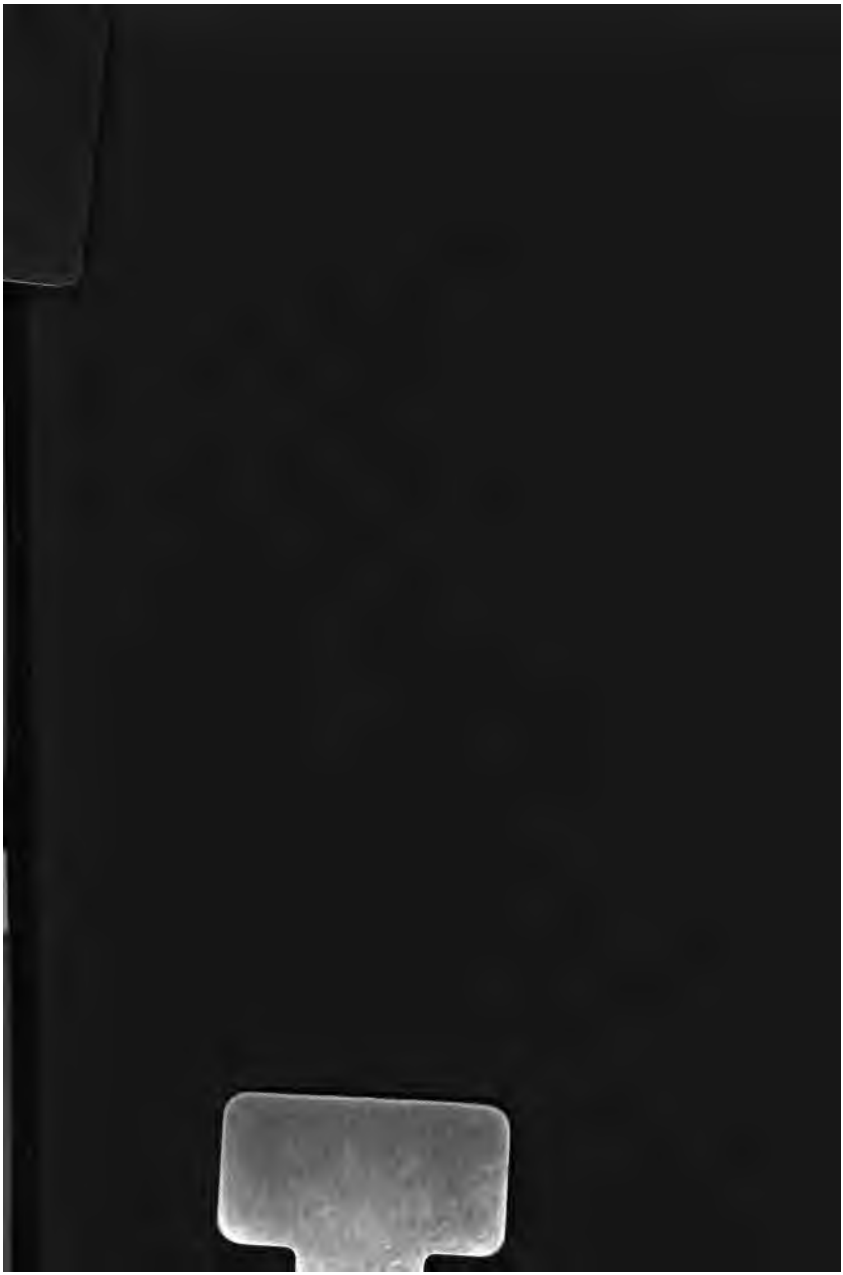
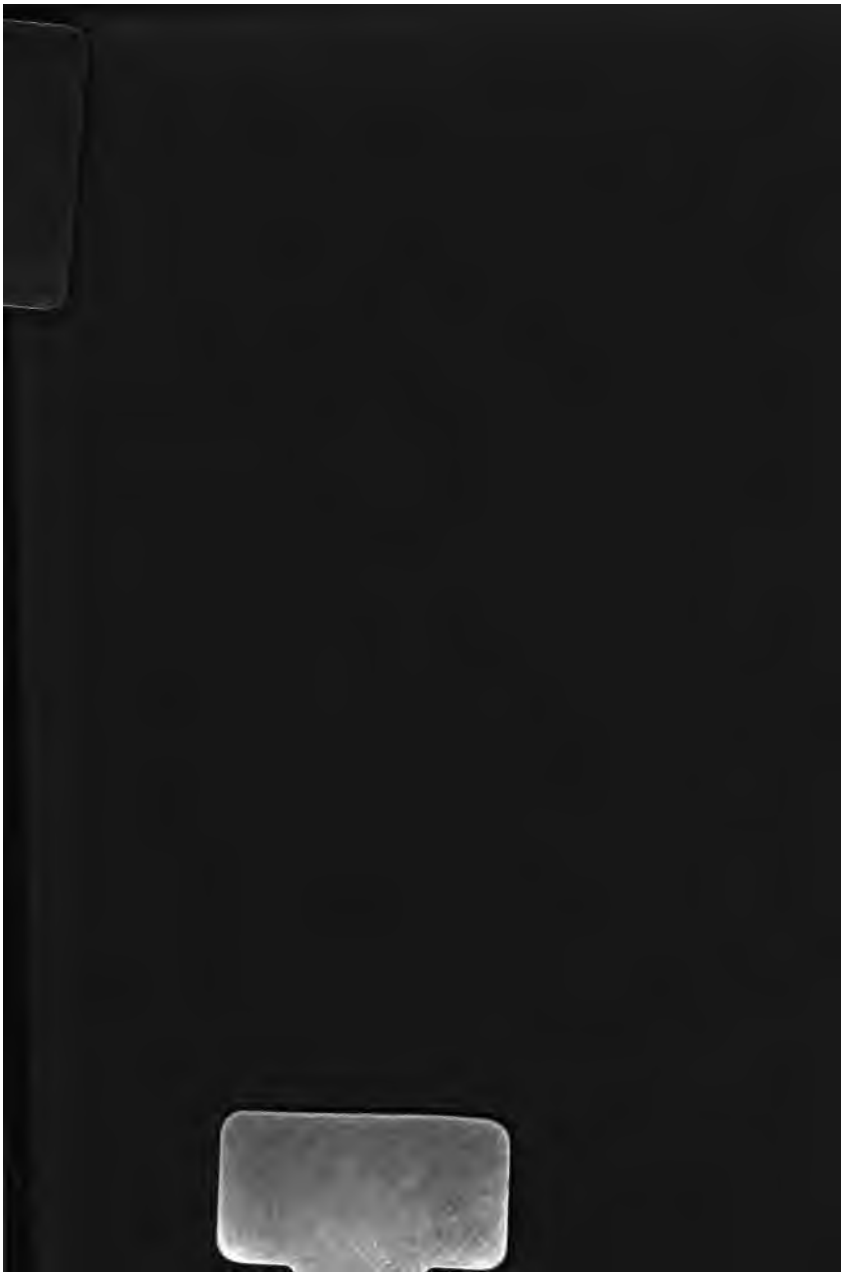
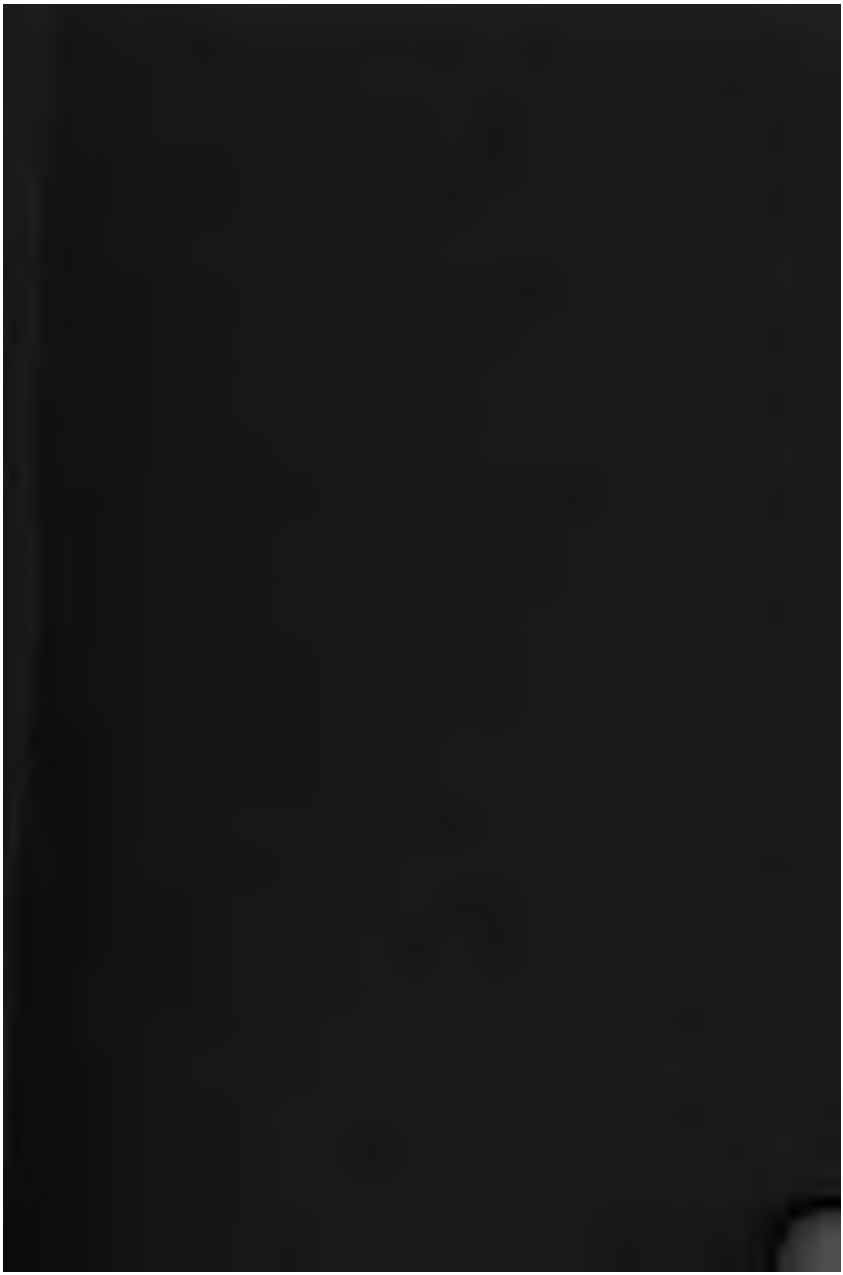






KEY TO MERRIFIELD'S

TECHNICAL ARITHMETIC

AND

MENSURATION.

LONDON: PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET

KEY TO MERRIFIELD'S

TECHNICAL ARITHMETIC

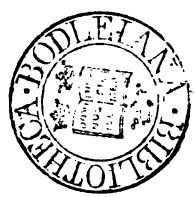
AND

MENSURATION.

BY THE

REV. JOHN HUNTER, M.A.

One of the National Society's Examiners of Middle-Class Schools:
Formerly Vice-Principal of the Society's Training College, Battersea.



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KEY

TO

TECHNICAL ARITHMETIC AND MENSURATION.

MIXED ADDITION AND SUBTRACTION.

(Page 18.)

(1) $8608 - 6435 = 2173$

(2) $358433 - 297543 = 60890$

(3) $5592405 - 2796202 = 2796203$

MULTIPLICATION.

(Page 24.)

(1) 19×17
133
323

(2) 98×17
686
1666

(3) 908×17
6356
15436

(4) 98×107
686
10486

(5) 9080×1070
635600
9715600

(6) 755×355
3775×7
26425
268025

(7)
$$\begin{array}{r} 38175 \times 34015 \\ 190875 \\ 152700 \\ 114525 \\ \hline 1298522625 \end{array}$$

(8)
$$\begin{array}{rl} 63245553 & \\ 31622777 & \\ \hline 442718871 & = \text{A} \\ 4869907581 & = 11\text{A} \\ 126491106 & \\ 126491106 & = \text{B} \\ 1011928848 & = 8\text{B} \\ 189736659 & \\ \hline 2000000018760681 & \end{array}$$

(9)
$$\begin{array}{r} 91 \times 11 \\ 91 \\ \hline 1001 \end{array}$$

(10)
$$\begin{array}{r} 42 \times 120 \times 720 \\ 5040 \\ \hline 288 \\ 360 \\ \hline 3628800 \end{array}$$

(11)
$$323323 \times 30 = 9699690$$

(12)
$$\begin{array}{r} 47712 \times 30102 \\ 95424 \\ 143136 \\ \hline 1436226624 \end{array}$$

(13)
$$\begin{array}{r} 4771213 \\ 602060000 - 1 \\ \hline 28627278 \\ 9542426 \\ 28627278 \\ \hline 2872556498780000 \\ 4771213 \\ \hline 2872556494008787 \end{array}$$

(14)
$$\begin{array}{rl} & 812358 \\ & 857 \\ \hline \text{A} = & 6498864 \\ 7\text{A} = & 45492048 \\ & 696190806 \\ & 2793 \\ \hline & 2088572418 \\ & 6265717254 \times 3 \\ & 18797151762 \\ \hline & 19444609211 58 \end{array}$$

(15)

744615
427282
5212305 = A
31273830 = 6A
20849220 = 4A
1489230
318160586430
15905
1590802932150
2863445277870
4772408796450
506034412716915O

(16)

930937 × 708421 = 6516559 = A
78198708 = 12A
19549677 = 3A
659495320477

659495320477 × 461762 = 1318990640954 = B
10551925127632 = 8B
10551925127632 = 8B
3956971922862
263798128190 8
304529878174100474

304529878174100474 = C
972744
2436239025392803792 = 8C = D
730871707617841137 6 = 3D = E
219261512285352341 28 = 3E
219261512285352341 28 = 3E
2740768903566904266
296229611814587191480656

(17)

1010101 × (1000000 − 1) × 9090909 × 1111111
99999999
99999999
99999999
101010089898 99 × 1010101
101010089898 99
101010089898 99
101010089898 99
1000000 × 10203039281705969799
10203039281705969799000000
10203029078666688093030201

(18)

$$3543 \times 2337$$
$$3543$$
$$10629$$
$$10629$$
$$24801$$
$$8279991$$

$$9827 \times 3616$$
$$58962$$
$$353772$$
$$35534432 \times 5501$$
$$177672160$$
$$177672160$$
$$19547491043 2$$

$$19547491043 2 \times (8280000 - 9)$$
$$8280000$$
$$156379928345 6$$
$$39094982086 4$$
$$156379928345 6$$
$$16185322583769600 0$$
$$175927419388 8$$
$$16185304991027661 12$$

(19) $(10000-2)(10000-1) \times 10000 \times (10000+1)(10000+2)$
$= (10000^2-2^2)(10000^2-1^2) \times 10000$;

$$100000000 - 4$$
$$100000000 - 1$$
$$10000000000000000 + 4$$
$$- 500000000$$
$$9999999500000004 \times 10000 = \textit{Ans.}$$

(20)

$$99990000$$
$$9999$$
$$9998000100 00$$
$$99980001$$
$$9997000299990000$$
$$99970002999 9$$
$$9996000599960000 10000$$
$$99960005999600 01$$
$$9995000999900004999900 00$$
$$99950009999000049999$$
$$9994001499800014999400 01$$

(21) 999700029999 × (100 mill. − 20003 + 1)

999700029999000000000
999700039996000299999
1999400059998000 0
999500099990000499999 × (10000 − 1)
9995000999900049999 9
999400149980001499940001

(22) $1234^4 = 1522756^2 = 2318785835536$

(23) 2345 × 1234 = 2893730; which × 3456
= 10000730880; and this × 4567 gives
45673337928960

(24) If the Rule of Division may here be anticipated, we have 123456789 = 111111110 + 9 − 1, and may work as follows :—

9876543210
10864197531
10864197531
10864197531
10864197531
9.) 109739368902606310
12193263210028959 0
98765432 1
12193263111263526 9

DIVISION.

(Page 30.)

(10) Cancelling 10 from dividend and divisor, we have the quotient of (57540547800088 8 + 9000000) ÷ 71836700 = 100th part of the former quotient = 8009909.

(11)

$$512 \begin{cases} 8)\,1000000000 \\ 8)\,125000000 \\ 8)\,15625000 \\ \hline 1953125 \end{cases}$$

Note.—The quotient for 25 may be taken as 4 times the 100th part of the dividend; that for 625 as 16 times the 10000th part; that for 1953125 as] 512 times the 1000-millionth part. Compare the above division by 512.

(12) Divide by $(8 \times 9 \times 9)$.

REDUCTION.

(Page 39.)

1. £137 9*s.* 11*d.*
   20
   2749*s.*
   12
   32999*d.*
   2
   65998 *hf. d.*

2. 120 far.) 21711 far.
   18 hf. cr + 111 far.

3. 145 tons 14 cwts. 3 qrs. 8 lbs.
   20 — 28
   112 × 2914 cwts. — 92 lbs.
   34968
   326460 lbs. × 16 = 5223360 oz.

4. 16) 1000000
   28) 62500
   4) 2232 qrs. 4 lbs.
   20) 558 cwts.
   27 t. 18 cwts. 4 lbs.

5. 4) 1870 years 273 da.
   467
   14 { centurial yrs. not leap.
   453 leap years
   273
   1870 × 365 + 726 = 683276 da.
   = 97610 wks. 6 da.

6. 365 da. × 24 × 60 × 60
   = 31536000 sec.

7. 60) 10000000
   60) 166666 min. 40 sec.
   24) 2777 hrs. 46 min.
   115 da. 17 hrs.

8. 157 lbs. 7 oz. 8 dwts.
   12
   1891 oz.
   20
   37828 dwts.
   24
   907872 grs.

∴ 0 yrs. 115 da. 17 hrs. 46 min. 40 sec.

9. 78 lbs. 10 oz. 5 drs.
12
946 oz.
8
7573 drs.
60
454380 grs.

10. 1000000 grs. ÷ 5760 grs.
= 173 lbs. 3520 grs.

11. 18 mi. 3 fur. 1 po.
8
147 fur.
40
5881 po. × 11
5881
2) 64691
32345½ yds.

12. 3) 100000 ft.
5½) 33333 yds. 1 ft.
11) 66666
40) 6060 po. 3 yds.
8) 151 fur. 20 po.
18 mi. 7 fur.
∴ 8 mi. 7 fur. 20 po. 10 ft.

13. 180° × 60 × 60
= 648000″

14. 5280 × 5280 = 27878400 sq. ft.

15. 1728 × 27 = 46656 cu. in.

16. 1000000 ÷ 1728 = 578 cu. ft. 1216 in.

17. 4) 444912 far.
12) 111228*d.*
21) 9269*s.* 8*s.*
441 gui.

18. 1000 gui.
42
5) 42000 sixp.
8400 hf. cr.

19. 18 gal. × 4 × 4 = 288 hf. pts.

20. 143 ac. 2 ro. 37 po.
4
574 ro.
40
22997 po.
30¼
5749¼
689910
6956591¼ yds.
9
62609331¼ ft.

21. 30¼) 14695 sq. yds.
121) 58780
40) 485 po. 23¾ yds.
12 ro. 5 po.
or 3 ac. 5 po. 23¾ yds.

22. $\frac{21000 \times 5760}{7000}$
= 3 × 5760 = 17280 lbs. av.

23. $\frac{1000}{16}$ lb. $\times \frac{7000}{1728}$ grs. $= \frac{437500}{1728} = 253\frac{76}{432}$ grs.

24. 108 gal. $\times 4 \times 4 = 1728$ hf. pts.

25. 2240 lbs. $\div$ 10 lbs. $= 224$ gal.

$$\frac{227 \times 224}{1728} = 35 \text{ cu. ft. } 1568 \text{ in.}$$

COMPOUND ADDITION.

(Page 43.)

11. 1000 t. + 50 t. + 12 t. 10 cwts. + 8 cwts. 3 qrs. 20 lbs. + 2 qrs. 6 lbs. 8 oz. = 1062 t. 19 cwts. 1 qr. 26 lbs. 8 oz.

12. 10 mi. + 12 mi. 4 f. + 3 mi. 1 f. + 5 mi. 5 f. 18 po. 1 yd.
+ 18 mi. 7 f. 20 po. 3 yds. 1 ft.
+ 15 mi. 6 f. 10 po. 2 yds. 2 ft. 4 in.
= 66 mi. 9 po. $1\frac{1}{2}$ yd. 4 in. = 66 mi. 9 po. 1yd. 1 ft. 10 in.

13. 10 ac. + 18 ac. 5 ch. 460 yds. + 2 ch. 358 yds. 3 ft.
+ 1 ch. 287 yds. 5 ft. 64 in. = 29 ac. 137 yds. 1 ft. 64 in.

14. 13 c. yds. + 5 c. yds. 22 ft. + 5 c. ft. 1360 in. = 19 c. yds. 1360 in.

15.

	£	s.	d.		£	s.	d.
A,	55364	14	10	*a*,	904411	5	11
B,	366182	3	8	*b*,	226504	13	9
C,	446737	4	10	*c*,	50496	6	1
D,	361640	14	11	*d*,	99636	16	8
E,	103962	2	1	*e*,	42673	2	1
				f,	5702	5	1
				g,	4462	10	9
£	1333887	0	4	£	1333887	0	4

COMPOUND SUBTRACTION.

(Page 46.)

3. 1873 yrs. 168 da. to June 17, 1873;
41 260
1831 yrs. 273 da. to September 30, 1831.

4. *See* Answer in Text-Book.

7. £12 1*s.* 6*d.* earned—£5 13*s.* 2*d.* spent = £6 8*s.* 4*d.* richer.

8. Only in the first two days did the spendings exceed the earnings; and the excess was 21*s.* 10*d.*+11*s.* 6*d.* = £1 13*s.* 4*d.*

9. 34 mi. 210 yds. E.—23 mi. 410 yds. W. = 10 mi. 1560 yds. E.

COMPOUND MULTIPLICATION.

(Page 48.)

3. 1341 cwts. 1 qr. 14 lbs.+1673 cwts. 3 qrs. = 150 tons 15 cwts. 14 lbs.

4. 2*s.* × 2845 = 5690*s.* 0*d.*
1*d.* × 2845 = 237 1
5452*s.* 11*d.* = £272 12*s.* 11*d.*

5. 149 lbs. × 861 = 128289 lbs. = 57 tons 5 cwts. 1 qr. 21 lbs.

6.

tons	cwts.	qrs.	lbs.	oz.
5	17	3	17	5
				43
253	9	3	16	7

7.

lbs.	oz.	dwts.	grs.
4	8	17	18
			12
56	10	13	0
			2
113	9	6	0

8.

mi.	f.	yds.
19	3	105
		17
330	3	25

9.

ac.	ro.	po.
579	3	23
		19
11017	3	37

10. 179

17 sq. ft.	9 in.
11	27

1253
179
3054 sq. ft. 27 in.

11.

yds.	qrs.	na.
18	2	3
		14
261	2	2

12.

cu. yd.	ft.	in.
1	15	1124
		7
11	1	956

13. $\frac{44 \text{ yds.} \times 1235}{1760 \text{ yds.}} = \frac{1235}{40}$ mi. = 30 mi. 7 fur.

14. 125375
193½ in.
62687½
376125
2382125
12) 24260062½ in.
660) 2021671 ft. 10½ in.
8) 3063 fur. 91 ft.
382 mi. 7 fur.
or, 382 mi. 7 fur. 91 ft. 10½ in.

15. The edge of the cog wheel turns through 150 half-inches, or 75 inches, per minute, = 4500 inches, or 375 feet, per hour.

16. 40 × 6 × 6 × 6 × 6 × 6 × 60 = 1866240 rev.

17. $\frac{7 \text{ ft.} \times 60 \times 60 \times 8}{5280 \text{ ft.}} = \frac{7 \times 60}{11}$ mi. = 38 mi. 320 yds.

18. 53 oz. × 6000 = 318000 oz. = 19875 lbs.
28) 19875 lbs.
709 qrs. 23 lbs. = 8 tons 17 cwts. 1 qr. 23 lbs.

19. $5280 \times 5280 \times 9 = 250905600$ oz.
$= 15681600$ lbs. $= 560057$ qrs. 4 lbs.
$= 140014$ cwts. 1 qr. 4 lbs. $= 7000$ tons 14 cwts. 1 qr. 4 lbs.
also, 250905600 *hf. d.* $= 10454400s. = £522720$

20. 66000000 oz. $= 4125000$ lbs. $= 147321$ qrs. 12 lbs.
$= 36830$ cwts. 1 qr. 12 lbs. $= 1841$ tons 10 cwts. 1 qr. 12 lbs.
also, £1 7*s.* 8*d.* $\times 10 \times 10 \times 10 =$ £1383 6*s.* 8*d.*

COMPOUND DIVISION.

(Page 51.)

3. $\frac{2240 \text{ lbs.} \times 40}{30000} = \frac{224}{75}$ lbs. $= 2$ lbs. $15\frac{59}{75}$ oz.

4. $\frac{16 \text{ oz.} \times 2240}{5280} = \frac{224}{33} = 6\frac{26}{33}$ oz.

7. 6 bottles $\times 108 = 54$ dozen; $\therefore 1350s. \div 54 = 25s.$

8. $\frac{45 \times 75}{4\frac{1}{2}} = 10 \times 75$ ft. $= 250$ yds. at 5*s.* 6*d.* $=$ £68 15*s.*

9. $\frac{5280 \text{ ft.} \times 167}{132 \text{ ft.}} = 40 \times 167$, or 6680

10. $\frac{16 \text{ oz.} \times 2240}{457 \text{ oz.}} = 78$ to the ton;

$$\frac{16 \times 112 \times 3465}{457} = \frac{6209280}{457} = 13587$$

11. 25 ft. $\times 16 \times 112 \div 5280 = 8$ mi. $853\frac{1}{3}$ yds.

12. $\frac{5280 \text{ ft.} \times 4}{27\frac{1}{2}} = \frac{5280 \times 8}{55} = 96 \times 8 = 768$

13. 296 ft. = 3552 in. = 28416 eighths of an inch;
28416 ÷ 625 = 45 eighths nearly, or $5\frac{5}{8}$ in.

14. $\frac{5280 \text{ ft.} \times 17}{6000} = \frac{88 \times 17}{100} = 14\frac{24}{25}$ ft.

15. $\frac{5280 \text{ ft.} \times 7\frac{1}{2}}{8347 \times 2} = \frac{19800}{8347}$ ft. = 2 ft. $4\frac{1}{2}$ in. nearly

16.
$$\begin{array}{r} £4\ 3s.\ 6d.) \\ \underline{40} \\ 167 \end{array} \qquad \begin{array}{r} £14350 \\ \underline{40} \\ )\ 574000 \\ \hline 3437 \text{ ac. } 20 \text{ po.} \end{array}$$

17. $\frac{5280 \text{ ft.} \times 60}{1435} = \frac{63360}{287} = 220$ ft. 9 in.

18. 73 × 119 = 8687 daily rations;
108044 lbs. 9 oz. ÷ 8687 = 12 lbs. 7 oz.

19. 360° × 60′ = 21600′;
$\frac{5280 \text{ ft.} \times 12315}{21600} = \frac{11 \times 821}{3} = 3010$ ft. 4 in.

20. $\frac{60 \times 12 \times 10 \times 12 \times 90}{9 \times 9 \times 8} = 200 \times 60 = 12000$

FRACTIONS.—Part I.

(Page 62.)

4. (4) $1\frac{3}{77} \times 13 \times 77 = 13 \times 13 = 169$

(5) $\frac{137}{137 \times 73} \times 73 \times 2 = 2$

(6) $\frac{1001}{137\times73}\times(137\times73)\times5 = 1001\times5 = 5005$

(7) $\frac{10001}{1001\times1001}\times(77\times13)\times(77\times13)\times289$
$= 10001\times289 = 2890289$

5. (4) $\frac{13}{77}\times\frac{1}{13\times77} = \frac{1}{5929}$ (6) $73\div146 = \frac{1}{2}$

(7) $\frac{77}{13\times13\times77} = \frac{1}{169}$ (8) $77\div77 = 1$

(9) $\frac{(77\times13)\times(77\times13)}{1001\times1001}\times\frac{17\times17}{1001} = \frac{289}{1001}$

(10) $\frac{334798}{91\times5117} = \frac{458\times731}{91\times7\times731} = \frac{458}{637}$

7. (1) £27 14*s.* 0*d.*
$\frac{1}{4}$ = 6 18 6
£20 15 6

(2) £31 10*s.*
5
7)157 10
£22 10

(3) $\frac{2}{9}$ of £6 9*s.*
9
2)58 1
£29 0 6

(4) $\frac{3}{7}$ of 35 cwts. = 5 cwts. × 3 = 15 cwts.

(5) $\frac{7}{16}$ of 3 tons = $\frac{1}{16}$ of 21 tons = 1 ton 6 cwts. 1 qr.

(6) $\frac{5}{64}$ of 96 cwts. = $\frac{5}{2}$ of 3 cwts. = $7\frac{1}{2}$ cwts.

(7) $\frac{5}{1288}$ of 75 cwts. = $\frac{1}{1288}$ of 525 cwts.
= 4 cwts. 11 lbs. 6 oz.

(8) $\frac{5}{11}$ of $\frac{1}{2}$ mi. = $\frac{5}{11}$ of 880 yds. = 400 yds.

(9) $\frac{47}{1210}$ of 640 ac. = $\frac{1}{1210}$ of 30080 ac.
= $\frac{1}{11}$ of 2734 ac. 5 ch. 220 yds. = 248 ac. 5 ch. 460 yds.

(11) $\frac{13\times53\times20\times24}{4032} = \frac{13\times53\times5}{42} = 82\frac{1}{42}$ grs.

(12) To find $\frac{63}{64}$ of 1 ton 14 cwts. 3 qrs. 0 lbs. 0 oz.
Subtract $\frac{1}{64}$ = 2 4 13
1 ton 14 cwts. 0 qrs. 23 lbs. 3 oz.

8. (6) $\frac{1001}{81} \times \frac{729}{91} = \frac{91 \times 11 \times 9 \times 81}{81 \times 91} = 11 \times 9 = 99$

(7) $\frac{33}{365} \times 14\frac{73}{1183} = \frac{33}{73} \times \frac{3327}{1183} = \frac{109791}{86359}$

9. (8) $\frac{19}{11 \times 17} \times \frac{17}{341} \times \frac{11}{19} = \frac{1}{341}$

(9) $\frac{39}{19} \times \frac{44}{19} \times \frac{43 \times 2}{37} \times \frac{19 \times 4}{43} \times \frac{9}{11} \times \frac{11}{13} \times \frac{17}{21} \times \frac{19}{21}$, &c.

12. (1) Instead of reducing the given fractions to a common denominator, we may first divide the denominators severally by their numerators, which gives—

$1\frac{6}{19}$, $1\frac{1}{3}$, $1\frac{5}{16}$, $1\frac{4}{13}$, $1\frac{3}{10}$, $1\frac{2}{7}$, $1\frac{1}{4}$, $1\frac{7}{22}$, $1\frac{8}{25}$, $1\frac{9}{28}$.

Now the *greatest* of these must belong to the *smallest* of the given fractions; therefore, neglecting the common integer 1, and reducing the annexed fractions to their common *numerator*, 2520, the greatest of them will have the least denominator. The new denominators, in order, are 7980, 7560, 8064, 8190, 8400, 8820, 10080, 7920, 7875, and 7840. Hence, the least of the given fractions is the 2nd, and the order of magnitude is the 2nd, 10th, 9th, 8th, 1st, 3rd, 4th, 5th, 6th, 7th. This method often saves much labour of calculation.

(2) Dividing denominators by numerators gives $3\frac{1}{8}$, $3\frac{1}{9}$, $3\frac{2}{17}$, $3\frac{3}{28}$, $3\frac{3}{25}$, $3\frac{3}{28}$. Neglecting the common integer 3, we have a common multiple of these *numerators* = 6, for which the corresponding denominators, in the given order, are 48, 54, 51, 52, 50, and 56. Hence the first given fraction is the least; and the order of magnitude of the given fractions is, 1st, 5th, 3rd, 4th, 2nd, 6th.

PROPORTION.

(Page 75.)

1. 365 da. : 219 da. :: 292*s.* : $\frac{292s. \times 3}{5} = 175\frac{1}{5}s.$
$= £8\ 15s.\ 2\frac{1}{2}d.$

2. 7 da. : 92 da. :: 18*s.*
or, 1 : $13\frac{1}{7}$ :: 54
2 6
236*s.* 6*d.* $= £11\ 16s.\ 6d.$

3. 18*s.* : 2000*s.* :: 7 da. : 7000 da. $\div 9 = 777\frac{7}{9}$ da.

4. 8 lbs. : 336 lbs. :: $3\frac{1}{6}s.$: $3\frac{1}{6}s. \times 6 \times 7 = £6\ 13s.$

5. 11 yds. : 95 yds. :: 42*d.* : 3990*d.* $\div 11 = 30s.\ 2\frac{3}{4}d.$

6. 15 tons : 27 tons :: $1\frac{1}{4}$ ton : $\frac{5\text{ t.} \times 9}{5 \times 4} = 2\frac{1}{4}$ tons.

7. 9 parts : 8 parts :: 10 lbs. × 19 : 1520 lbs. $\div 9$
$= 168\frac{8}{9}$ lbs.

8. 15 ft. : 65 ft. :: 13 ft. : $\frac{13\text{ ft.} \times 13}{3} = 56\frac{1}{3}$ ft.

9. 25 cop. : 18 cop. :: 35*s.* : $\frac{7s. \times 18}{5} = 25s.\ 2\frac{1}{2}d.$

10. 1000 shr. : 4840 × 2 shr. :: $13\frac{1}{2}s.$: $\frac{27s. \times 484}{100}$
$= £6\ 10s.\ 8d.$

11. $\frac{3}{4}$ yd. wide : 9 yds. wide :: 9 yds. long : 108 yds. long.

12. For 150 she gives 2*s.* 8*d.* + 8*d.*, and gets 4*s.* 6*d.* + 2*s.* 3*d.*, therefore on 150 her profit is 6*s.* 9*d.* − 3*s.* 4*d.* = 3*s.* 5*d.*; hence, 3*s.* 5*d.* : 10*s.* :: 150 : 439.

13. 59 lbs. of the metal measure 195 cu. in.; hence, 195 : 1728 :: 59 lbs. : 522 lbs. 13 oz.

14. 45 *hf. d.* : 112 *hf. d.* :: 141 yds. : $350\frac{14}{15}$ pairs.

15.

16 : 3 :: 7 cwts. 3 qrs. :	1 cwt.	1 qr.	22 lbs.	12 oz.	lead,
„ : 5 :: „ „ :	2	1	19	4	tin,
„ : 8 :: „ „ :	3	3	14	0	bism.
16	7	3	0	0	

16. 3600″ : 1″ :: 12 in. × 364441 : $1214\frac{2}{3}$ in. very nearly.

17. 23 hrs. 56 min. 4 sec. : 24 hrs. :: $365\frac{1}{4}$ da. : $\frac{365\frac{1}{4} \times 24 \times 60 \times 60}{86164} = \frac{1461 \times 90 \times 60}{21541} = 366\frac{1}{4}$ sid. da. very nearly.

18. $\frac{3}{4} \times \frac{3}{4}$: $1\frac{1}{4} \times 1\frac{1}{4}$ :: 20 ft. : $55\frac{5}{9}$ ft.

19. $42 \times 18 \times 12$: 24×9 :: 2 in. : $\frac{1}{21}$ in.—See remark at foot of page 281 of the 'Arithmetic.'

20. 33000 lbs. ÷ 10 lbs. = 3300 gal. water;

840 ft. : 1 ft.
1 h. p. : 120 h. p.
1 min. : 60 min. × 24

} :: 3300 gal. : $\frac{3300 \times 60 \times 24}{7}$ = $678857\frac{1}{7}$ gal.

21. $11\frac{4}{10}$: 21 :: 112 lbs. : 3920 lbs. ÷ 19 = $206\frac{6}{19}$ lbs.

22. $11\frac{4}{10}$: 21 :: 1200 cu. in. : $2210\frac{10}{19}$ cu. in.

23. 100 gal. P. S. : $54 \times \frac{33}{47}$ P. S. :: 49 gal. alc. : $\frac{49 \times 54 \times 33}{47 \times 100} = \frac{87318}{4700} = 18\frac{1359}{2350}$ gal.

24. 1825 m. : 15000 m.
60 da. : 182½ da. } :: 73 t. 9 cwts. 2 qrs. 16 lbs.

$\frac{15000 \times 365}{3650 \times 60} = 25$;

5
367 8 0 24
5
1837 t. 1 cwts. 0 qrs. 8 lbs.

25. 1 sec. : $3\frac{4}{7}$ sec. :: 32 ft. : $114\frac{2}{7}$ ft.

26. The spaces passed through are as the squares of the times; hence,
128 : 1600 :: 3^2 : $112\frac{1}{2}$; and $\sqrt{112\frac{1}{2}} = 10\frac{3}{5}$ sec. nearly.

27. The coach has gone $8\frac{1}{2} \times 1\frac{1}{4}$ mi. $= 10\frac{5}{8}$ mi. before the horseman starts. He gains upon the coach at the rate of $14 - 8\frac{1}{2}$, or $5\frac{1}{2}$, miles per hour; and to overtake the coach he has to gain $10\frac{5}{8}$ mi.; hence,

$5\frac{1}{2}$ mi. : $10\frac{5}{8}$ mi. :: 1 hr. : $\frac{85}{44}$ hrs.
$\frac{85}{44}$ hrs. at 14 mi. an hour $= \frac{595}{22}$ mi. = 27 mi. 80 yds.

28. $3\frac{3}{7}$: $5\frac{5}{6}$ :: £5 : $\frac{£5 \times 35 \times 7}{24 \times 6} = £8$ 10*s.* $1\frac{2}{3}$*d.*

29. $\frac{38}{70}$: $\frac{29}{70}$ :: 1728 cu. in. : $\frac{864 \times 29}{19}$ cu. in.
$= 1318\frac{14}{19}$ cu. in.

30. $\frac{3}{4}$ of $\frac{2}{15}$, or $\frac{1}{10}$ of the whole, weighs $2\frac{1}{3} \times \frac{57600}{70000}$ lb. av. $= \frac{48}{25}$ lb. av.; hence,

$\frac{1}{10}$: $\frac{37}{52}$ of $\frac{13}{15}$ :: $\frac{48}{25}$ lb. av., or 6 : 37 :: $\frac{48}{25}$ lb. av.
: $11\frac{21}{25}$ lbs. av.

31. The weight of magnesia in 1 cu. ft. of sea-water
$= \frac{11}{180}$ of $\frac{8}{250}$ of $\frac{1}{35}$ of a ton; which, reduced to grains,
$= \frac{11}{180} \times \frac{8}{250} \times \frac{1}{35} \times 2240 \times 7000 = \frac{11}{5} \times 448 = 985\frac{3}{5}$ grs.

PRACTICE.

(Page 82.)

1. 10000 @ $8\frac{1}{2}d.$

$4d.$	$\frac{1}{3}$	3333*s.*	$4d.$
„	„	3333	4
$\frac{1}{2}d.$	$\frac{1}{8}$	416	8
		20) 7083	4
		£354	3*s.* 4*d.*

2. 7895 @ (1*s.* − $1\frac{3}{4}d.$)

$1\frac{1}{2}d.$	$\frac{1}{8}$	986*s.*	$10\frac{1}{2}d.$
$\frac{1}{4}d.$	$\frac{1}{6}$	164	$5\frac{3}{4}$
subtract		1151	$4\frac{1}{4}$
		20) 6743	$7\frac{3}{4}$
		£337	3*s.* $7\frac{3}{4}d.$

3. 4) 1000000 @ (1*s.* − $\frac{1}{4}d.$)

12) 250000

subtract 20833*s.* 4*d.*

20) 979166 8

£48958 6*s.* 8*d.*

4. 31416 @ 2*s.* 11*d.*

2*s.* 6*d.*	$\frac{1}{8}$	3927	
5*d.*	$\frac{1}{6}$	654	10
		£4581	10*s.*

5. 21828 @ 14*s.* 9*d.*

10*s.*	$\frac{1}{2}$	10914	
4*s.*	$\frac{1}{5}$	4365	12
6*d.*	$\frac{1}{8}$	545	14
3*d.*	$\frac{1}{2}$	272	17
		£16098	3*s.*

6. 12) 1745 @ (£1 − 1*d.*)

20) 145*s.* 5*d.*

subtract 7 5 5

£1737 14*s.* 7*d*

7. 57712 @ 16*s.*

8

£46169 12*s.*

8. 43424 @ 17*s.*

10*s.*	$\frac{1}{2}$	21712	
5*s.*	$\frac{1}{2}$	10856	
2*s.*	$\frac{1}{5}$	4342	8
		£36910	8*s.*

9. 30103 @ (£1 − 1*s.* 6*d.*)

1*s.*	$\frac{1}{20}$	1505	3
6*d.*	$\frac{1}{2}$	752	11 6
subtract		2257	14 6
		£27845	5*s.* 6*d.*

10. 47712 @ 16*s.* $10\frac{1}{2}d.$

10*s.*	$\frac{1}{2}$	23856	
6*s.* 8*d.*	$\frac{1}{3}$	15904	
2*d.*	$\frac{1}{48}$	397	12
$\frac{1}{2}d.$	$\frac{1}{4}$	99	8
		£40257	0*s.*

11. 1760 @ 3*s.* 11*d.*

1*s.*	$\frac{1}{20}$	88	
2*s.* 6*d.*	$\frac{1}{8}$	220	
5*d.*	$\frac{1}{6}$	36	13 4
		£344	13*s.* 4*d.*

12. 2240 @ 8*s.* 7½*d.*

	¼	560
5*s.*	½	280
2*s.* 6*d.*	⅛	112
1*s.*	⅛	14
1½*d.*		£966

13. 5760 @ 1¾*d.*

1½*d.*	⅛	720*s.*
¼*d.*	⅙	120
		20)840
		£42

14. 43560 @ 1¼*d.*

1*d.*	1/12	3630*s.*
¼*d.*	¼	907 6
		20)4537 6
		£226 17*s.* 6*d.*

16.

		46656 @ 1 lb. 13 oz.
8 oz.	½	23328
4 oz.	½	11664
1 oz.	¼	2916
		28)84564 lbs.
		3020 qrs. 4 lbs.
		= 37 t. 15 cwts. 4 lbs.

15. 4851 yds @ 4*d.* a yd.

4*d.* | ⅓ | 1617*s.* = £80 17*s.*

17. 257354 @ 64 lbs.

32 lbs. | 1/70 | 3676 t. 9 cwts. 2 qrs. 24 lbs.

2

7352 t. 19 cwts. 1 qr. 20 lbs.

18. 37350 @ 1*s.* 5½*d.*

1*s.*	1/20	1867 10
4*d.*	⅓	622 10
1½*d.*	⅛	233 8 9
		£2723 8*s.* 9*d.*

19. 27000000 @ 5*d.*

4*d.*	1/60	450000
1*d.*	¼	112500
		£562500

20. 3754 @ £3 17*s.* 10½*d.*

3

		11262
15*s.*	¼	2815 10
2*s.* 6*d.*	⅛	469 5
3*d.*	1/10	46 18 6
1½*d.*	½	23 9 3
		£14617 2*s.* 9*d.*

21. $367\frac{1}{2}$ @ 6 lbs. 11 oz.

6

	2205 lbs.	
8 oz. $\frac{1}{2}$	183 lbs.	12 oz.
2 oz. $\frac{1}{8}$	45	15
1 oz. $\frac{1}{16}$	22	$15\frac{1}{2}$

28) 2457 lbs. $10\frac{1}{2}$ oz.

87 qrs. 21 lbs.

= 1 t. 1 cwt. 3 qrs. 21 lbs. $10\frac{1}{2}$ oz.

	lbs.	oz.	
	2457	$10\frac{1}{2}$	@ $3\frac{1}{2}d.$
$3d.$ $\frac{1}{80}$	£30	14	3
$\frac{1}{2}d.$ $\frac{1}{6}$	5	2	$4\frac{1}{2}$
$\frac{2}{3}$ of $3\frac{1}{2}d.$			$2\frac{1}{4}$

cost £35 16*s.* $9\frac{3}{4}d.$

22.

	15840	@ 189 lbs.
56 lbs. $\frac{1}{3}$	7920	
14 lbs. $\frac{1}{4}$	1980	
7 lbs. $\frac{1}{2}$	990	

26730 cwts.

= 1336 t. 10 cwts.

	26730	@ 8*s.* 10*d.*
3*s.* 4*d.* $\frac{1}{6}$	4455	
5*s.* $\frac{1}{4}$	6682	10
6*d.* $\frac{1}{10}$	668	5
	£11805	15*s.*

23.

	850	@ £9 8*s.* 6*d.*
	9	
	7650	
1*s.* $\frac{1}{20}$	42	10
5*s.* $\frac{1}{4}$	212	10
2*s.* 6*d.* $\frac{1}{2}$	106	5
	£8011	5*s.*

	1400	@ £5 11*s.* 6*d.*
	5	
	7000	
10*s.* $\frac{1}{2}$	700	
1*s.* $\frac{1}{10}$	70	
6*d.* $\frac{1}{2}$	35	
	£7805	

Difference = £206 5*s.*

24. 600 ft. @ 76 lbs. = 45600 lbs.
1800 ft. @ 18 lbs. = 32400

28) 13200

4) 471 qrs. 12 lbs.

Difference of weight = 117 cwts. 3 qrs. 12 lbs.

= 5 t. 17 cwts. 3 qrs. 12 lbs.

	32400	@ 23*s.* 6*d.*
1*s.* $\frac{1}{20}$	1620	
2*s.* 6*d.* $\frac{1}{8}$	4050	
	£38070	
	35340	

	45600	@ 15*s.* 6*d.*
10*s.* $\frac{1}{2}$	22800	
5*s.* $\frac{1}{4}$	11400	
6*d.* $\frac{1}{10}$	1140	
	£35340	

£2730 ÷ 112 = £24 7*s.* 6*d.* diff. of cost.

25. The given quantity = 512 tons 16 cwts. $3\frac{3}{4}$ qrs.
= 512 tons $16\frac{15}{16}$ cwts. = $512\frac{271}{320}$ tons; which at £1
= £512 16*s.* $11\frac{1}{4}$*d.*

					9
3*s.* 4*d.*	$\frac{1}{6}$	4615	12	$5\frac{1}{4}$
2*d.*	$\frac{1}{20}$	85	9	$5\frac{3}{4}$
		4	5	$5\frac{3}{4}$
		£4705	7*s.*	$4\frac{3}{4}$

26. 310838 yds. @ $3\frac{1}{2}$*d.*

3*d.*	$\frac{1}{80}$	£3885	9*s.*	6*d.*
2*d.*	$\frac{1}{6}$	647	11	7
		£4533	1*s.*	1*d.*

27. $4962300\frac{3}{4}$ sq. ft. @ 3*s.* 10*d.*

		£4962300	15*s.*	
3*s.* 4*d.*	$\frac{1}{6}$	827050	2	6
6*d.*	$\frac{1}{40}$	124057	10	$4\frac{1}{2}$
		£951107	12*s.*	$10\frac{1}{2}$*d.*

FRACTIONS.—(Part II.)

(Page 95.)

10. (1) $\frac{19039 \text{ f.}}{20003 \text{ f.}} \left(\div \frac{241}{241}\right) = \frac{79}{83}$

(2) $\frac{13792381 \text{ in.}}{32080621 \text{ in.}} \left(\div \frac{76201}{76201}\right) = \frac{181}{421}$

(3) $\frac{33452239 \text{ ft.}}{5806592401 \text{ ft.}} \left(\div \frac{76201}{76201}\right) = \frac{439}{76201}$

(4) $\frac{2521777 \text{ oz.}}{36268013 \text{ oz.}} \left(\div \frac{1259}{1259}\right) = \frac{2003}{28807}$

11. (1) $\frac{187+70}{187-70} - \frac{243+14}{54+63} = \frac{257}{117} - \frac{257}{117} = 0$

(2) $\frac{105+63+45}{126+90+70} \times \frac{13}{71} \times \frac{22}{3} = 1$

(3) $4+\frac{1}{5\frac{1}{5}} = 4\frac{5}{26}$; $3+\frac{1}{4\frac{5}{26}} = 3\frac{26}{109}$;

$2+\frac{1}{3\frac{26}{109}} = 2\frac{109}{353}$; $1+\frac{1}{2\frac{109}{353}} = 1\frac{353}{815}$

(4) $5+\frac{6}{7\frac{8}{9}} = 5\frac{54}{71}$; $3+\frac{4}{5\frac{54}{71}} = 3\frac{284}{409}$;

$1+\frac{2}{3\frac{284}{409}} = 1\frac{818}{1161}$

(5) $1+\frac{1}{1\frac{1}{2}} = 1\frac{2}{3}$; $1+\frac{1}{1\frac{2}{3}} = 1\frac{3}{5}$; $1+\frac{1}{1\frac{3}{5}} = 1\frac{5}{8}$;

$1+\frac{1}{1\frac{5}{8}} = 1\frac{8}{13}$

(6) $\frac{\frac{4}{10}+\frac{4}{17}+\frac{6}{37}}{\frac{3}{10}+\frac{4}{17}+\frac{5}{26}} = \frac{\frac{108}{170}+\frac{6}{37}}{\frac{91}{170}+\frac{5}{26}} = \frac{5016}{37\times170} \times \frac{26\times170}{3216}$

$= \frac{5016\times26}{3216\times37} = \frac{209\times13}{67\times37} = \frac{2717}{2479}$

12. (1) $\frac{34\times\frac{3}{4}}{17\times12} = \frac{2}{16} = \frac{1}{8}$

(2) $\frac{3}{11}$ yd. $\div 5\frac{1}{2}$ yds. $= \frac{3}{11} \times \frac{2}{11} = \frac{6}{121}$

(3) $\frac{3}{11}$ of $5\frac{1}{2}$ yds. $= \frac{3}{11}$ of $\frac{11}{2}$ yds. $= \frac{3}{2}$ of a yd.

(4) $\frac{\frac{9}{14} \text{ of } 7000 \text{ grs.}}{5760 \text{ grs.}} = \frac{450}{576} = \frac{25}{32}$

(5) $\frac{24 \text{ grs.}}{20 \text{ grs.}} = \frac{6}{5}$ (6) $\frac{7}{110}$ of $\frac{121}{4}$ sq. yd. $= \frac{77}{40}$ sq. yd.

(7) $(\frac{35}{1331}$ of 36 in. $\times$ 1760$) \div$ 1000000 in.

$= (\frac{7}{11}$ of $36 \times 16) \div 20000 = \frac{7 \times 9 \times 4}{11 \times 1250} = \frac{126}{6875}$

(8) $\frac{988027}{974153} = \frac{991 \times 997}{991 \times 983} = \frac{997}{983}$

(9) $\frac{844497 \text{ oz.}}{908267 \text{ oz.}} = \frac{911 \times 927}{911 \times 997} = \frac{927}{997}$

(10) $\frac{783221 \text{ yds.}}{774391 \text{ yds.}} = \frac{883 \times 887}{883 \times 887} = \frac{887}{877}$

(11) $20\frac{40}{88}$ cwts. $+\frac{47}{97}$ cwt. $+\frac{5224}{8633 \times 112}$ cwt.

$= 20$ cwt. $+\frac{8063}{8633}$ cwt. $+\frac{653}{8633 \times 14}$ cwt.

$= 20\frac{11753536}{120869}$ cwts.

(12) $\frac{3}{40}$ ton $+\frac{1}{56}$ ton $+\frac{351719}{29606 \times 2240}$ ton

$= \frac{208 \times 29606 + 351719}{29606 \times 2240} = \frac{6509767}{66317440}$ ton

13. (1) The minute hand, M, is exactly over the hour hand, H, at 12 o'clock, and will be over H again when it has gained 60 minute spaces on H. Now M gains on H 55 minute spaces per hour, and therefore will gain 60 minute spaces in $\frac{60}{55}$ or $1\frac{1}{11}$ hour, that is, at 1 hour $5\frac{5}{11}$ minutes. So the times required are at 1 hour $5\frac{5}{11}$ minutes, and at every successive interval of 1 hour $5\frac{5}{11}$ minutes after that, viz.—at $10\frac{10}{11}$ minutes past 2, $16\frac{4}{11}$ minutes past 3, &c.

(2) The first sluice takes off 1 measure per hour, the whole quantity in the reservoir being 7 such measures; and the second sluice takes off $\frac{7}{13}$ of a measure per hour; so the two together take off $1\frac{7}{13}$ measures per hour, or the 7 measures in $7 \div 1\frac{7}{13} = 91 \div 20 = 4$ hours 33 minutes.

DECIMAL FRACTIONS.

(Page 120.)

5. (2) $152 \div 5000 = \cdot 152 \div 5$ (3) $125 \div (16 \times 8 \times 8)$

(4) $256 \div 625000 = \cdot 256 \div 625 = 2 \cdot 048 \div 5000$
$= \cdot 002048 \div 5$

(5) $1001 \div (16 \times 16 \times 8 \times 8)$

(6) $375000 \div 1024 = 375000 \div (16 \times 8 \times 8)$

(7) $1137 \div 625 = 9096 \div 5000$

(8) $1000 \cdot 0000 \div 1048576$
$= 1000 \cdot 0000 \div (16 \times 16 \times 16 \times 16 \times 16)$

(9) $\cdot 008192 \div 244140625000$
$= \cdot 065536 \div 1953125000000$
$= \cdot 000000000524288 \div 15625 =$ &c.

(10) $32000 \div \cdot 001 = 32000000 \div 1$

(11) $10000000000000 \div 1$

(12) $320 \div 78125 = 2560 \div 625000$
$= 2 \cdot 56 \div 625 = 20 \cdot 48 \div 5000$

6. (2) $1810 \div 27$ (4) $\cdot 1 \div 1001$ (5) $1 \div 1111$

(6) $\cdot 00729 \div 59049 = \cdot 00081 \div 6561$
$= \cdot 00009 \div 729 = \cdot 00001 \div 81$

(7) $10000000 \div 11636 = 2500000 \div 2909$

(8) $100000 \div 85943689 = 100000 \div (13 \times 6611053)$

(9) $9921461 \div 78539$ (10) $78539 \div 9921461$

(11) $1599820000000 \div 9840018$

(12) $98400180 \div 159982$

7. (15) $\frac{41}{492} = \frac{1}{12} = \cdot 08\dot{3}$

9. (1) $\cdot 31\frac{6}{9} = 31\frac{2}{3} \div 100 = 95 \div 300 = \frac{19}{60}$

(2) $\cdot 1\frac{36}{99} = 1\frac{4}{11} \div 10 = 15 \div 110 = \frac{3}{22}$

(3) $\cdot 1\frac{63}{99} = 1\frac{7}{11} \div 10 = 18 \div 110 = \frac{9}{55}$.

(4) $\cdot 57\frac{75}{99} = 57\frac{25}{33} \div 100 = \frac{953}{1650}$

(5) $\cdot 5\frac{925}{999} = 5\frac{25}{27} \div 10 = \frac{16}{27}$

(6) $8\cdot 4\frac{84}{99} = 8 + (4\frac{28}{33} \div 10) = 8\frac{146}{330}$

(7) $\cdot 9\frac{142857}{999999} = \cdot 9\frac{1}{7} = 9\frac{1}{7} \div 10 = \frac{32}{35}$

(8) $\cdot 7\frac{2457}{9999} = 7\frac{1}{407} \div 10 = \frac{285}{4070}$

(9) $\cdot 2\frac{997543}{999999} = \cdot 2\frac{406}{407} = 2\frac{406}{407} \div 10 = \frac{132}{4070}$

(10) $\cdot 75\frac{362}{363} = 75\frac{362}{363} \div 100 = \frac{27587}{36300}$

10. (1)

2	1·0
3	·5
4	·16666667
5	·04166667
6	·00833333
7	·00138889
8	·00019841
9	·00002480
10	·00000276
	·00000028
	2·7182818

(2)

1·0		1·0
·5	÷ 3 =	·16666667
·04166667	÷ 5 =	·00833333
·00138889	÷ 7 =	·00019841
·00002480	÷ 9 =	·00000276
·00000028	÷ 11 =	·00000003
1·54308064	—	1·17520120

= ·3678794; which will be found to be equal to 1 ÷ 2·7182818

(3)

+	1·0
−	·16666667
+	·00833333
−	·00019841
+	·00000276
−	·00000003
+	1·00833609
−	·16686511
	·841471

(4)

+	·5
−	·04166667
+	·00138889
−	·00002480
+	·00000028
+	·50138917
−	·04169147
	·4596977

(5)
$$\begin{array}{l} \cdot 5 \\ \cdot 02083333 \\ \cdot 00130208 \\ \cdot 00009456 \\ \cdot 00000746 \\ \hline \cdot 5222374 \end{array}$$

(6)
$$\begin{array}{r} +\cdot 5 \\ -\cdot 00833333 \\ +\cdot 00041667 \\ -\cdot 00002639 \\ +.00000187 \\ -\cdot 00000014 \\ \hline +\cdot 50041854 \\ -\cdot 00835986 \\ \hline \cdot 4920587 \end{array}$$

13. (1) $\frac{1}{8} = \cdot 125$; $\frac{1}{20} = \cdot 05$; $\frac{1}{80} = 0\cdot 125$; $\frac{1}{240} = \cdot 00416\dot{6}$; $\frac{1}{960} = \cdot 0010416\dot{6}$; of £1.

(2) $\frac{1}{20} = \cdot 05$; $\frac{1}{80} = \cdot 0125$; $\frac{1}{2240} = \cdot 0004464286$; $\frac{1}{16}$ of $\frac{1}{2240} = \cdot 0000279018$; of a ton.

$\frac{1}{4} = \cdot 25$; $\frac{1}{112} = \cdot 008928571$; $\frac{1}{16}$ of $\frac{1}{112}$ $= \cdot 000558036$; of a cwt.

$\frac{1}{28} = \cdot 03571429$; $\frac{1}{16}$ of $\frac{1}{28} = \cdot 002232143$; of a quarter.

$\frac{1}{16} = \cdot 0625$ of a lb.

(3) 1 sq. in. $= \frac{1}{144}$ sq. ft. $= \frac{1}{144 \times 9}$ sq. yd.

$= \frac{1}{144 \times 9 \times \cdot 0484}$ sq. link

$= \frac{4}{144 \times 9 \times 121}$ sq. po., &c.; hence,

1 sq. in. $= \cdot 00694444444$ sq. ft.
$= \cdot 00077160494$ sq. yd.
$= \cdot 01594225079$ sq. lk.
$= \cdot 00002550760$ sq. po.
$= \cdot 00000159422$ sq. ch.
$= \cdot 00000063769$ ro.
$= \cdot 00000015942$ ac.
$= \cdot 00000000025$ sq. mi.

Similarly, 1 sq. ft. $= \frac{1}{9}$ sq. yd. $= \frac{1}{9 \times \cdot 0484}$ sq. link, &c.

(4) 1 degree $= \frac{1}{360} = \frac{1}{9}$ of ·025 = ·0027777778
1 minute $= \frac{1}{60}$ of a degree = ·0000462963
1 second $= \frac{1}{60}$ of a minute = ·0000007716

(5) 180) 3·1415926536
60) ·01745329252, length of the degree,
60) ·0002908882 1, „ of the minute,
·0000048481 4, „ of the second.

(6) 180° being equal to 3·1415926 times the radius, we have 180°÷3·1415926 = 57·29578°

14. (1) 12) 6·0*d.*
20) 17·5*s.*
£ ·875

(2) 12) 8·0*d.*
20) 16·666*s.*
£ ·833

(3) 12) 5·75*d.*
20) 9·48*s.*
£ ·474

(4) 12) 11·75*d.*
20) 19·979*s.*
£ 3·999

(5) 12) 7·25*d.*
20) 11·604*s.*
£ 5·5802

(6) 12) 9·5*d.*
20) 13·792*s.*
£ 4·6896

15. (1) 37·8593 mèt.÷·3047945 mèt. = 124·2125 ft.

(2) 19896 ft. × ·3047945 mèt. = 6064·19 mèt.

(3) 292971000 grains÷15.4323487 grains
= 1898421 3 grammes

(4) 3597·832 grammes × 15·43235 grains
= 55523 grains

(5) 1573 grains × ·064799 grammes
= 101·929 grammes

(6) 9038 ac. × ·4046711 hectare = 3657·42 hectares

(7) 395·47 ac.
4
1·88 ro.
40
395 ac. 1 ro. 35·2 po.

(8) A gallon = 277·274 c. in.
A litre = ·220096 gall.
Product, 61·0269 c. in.

(9) A kilogramme = 2·2046 lbs. av., and a gallon of water weighs 10 lbs.; hence, 10÷2·2046
= 4·536 kilogr.

(10) A cubic mètre = the bulk of a kilolitre
= 61027 c. in., which × 2·8735 gives 175361 c. in.
= 101 c. ft. 833 in.

16. (1) 16)5·0 oz.
28)17·3125 lbs.
4)3·618303 qrs.
20)14·904576 cwts.
·7452288 ton.

(2) 20)19·0 grs.
3)7·95 scr.
8)7·65 dr.
12)13·95625 oz.
1·16302 lb.

(3) 57·29578 deg.÷180 deg. = ·31831 of semicircumf.
·31831 × 3·14159 = 1 = radius

(4) 40)47 po.
4)4·175 ro.
640 { 8)348·04375 ac.
80)43·50547
4·543818 sq. mi.
2560
348·04375
2908·04375 ac.

(5) 12)1397 in.
12)116·4166
12)9·70139
21·80845 ft.
513
35 { 5)534·80845 ft.
7)106·96169
15·28024 tons

(6) 72635 grs.÷7000 grs. = 10·3764 lbs.

(7) 16)15·0 oz. 400·0 grs.
17·9375 × 7000 = 125562·5 grs.
24)125962·5 grs.
20)5248·4375 dwt.
12)262·421875 oz.
21·868489 lbs. Troy

(8)
$$\begin{array}{l} 40)\ 37{\cdot}0 \text{ po.} \\ \quad 4)\ \ 2{\cdot}925 \text{ ro.} \\ 11)\ 57{\cdot}7312 \text{ ac.} \times 36 \div 30\tfrac{1}{4}\text{;} \\ \quad\quad 5{\cdot}2483 \\ 11)\ \ 5{\cdot}2483 \\ \quad\quad\ \ {\cdot}4771 \\ \hline 68{\cdot}705 \text{ ac.} \end{array}$$

Note.—$\frac{36}{30\frac{1}{4}} = \frac{12 \times 12}{11 \times 11} = 1\frac{1}{11} \times 1\frac{1}{11}$

17. (1) 3·14159265 t. =62 cwts. 3 qrs. 9 lbs. 2·68 oz. *Ans.*

20	= 112594·68 oz. *Ans.*
62·83185300 cwts.	= 7037·1675 lbs.
4	7000
3·327412 qrs.	49260172·5 grs. *Ans.*
28	
9·167536 lbs.	
16	
2·680576 oz.	

(2) ·318309886 of 180° = 57·29578°
= 57° 17′ 44·8″ = 206264·8″

(3) ·43428793

365
158·50159 da.
24
12·36216 hrs.
60
21·7296 min.

(4) ·57721314987

8
4·61770519896
80
369·4164159168 ac.
43560
16091779·077336 ft.

(5) ·487956324 t.

35
17·07847134 cu. ft.
1728
135·5985 cu. in.

(6) 29511·5985 cu. in.
277·274 cu. in.

= 106·4348 gall.

18. (1) 1 lb. per foot = 3·2808992 lbs. per mètre
$$= \frac{3\cdot2808992}{2\cdot204621} \text{ kilogr.} = 1\cdot48819$$

(2) 1 kilogr. per mètre $= \frac{1}{3\cdot2808992}$ kilogr. per foot
$$= \frac{2\cdot204621}{3\cdot2808992} \text{ lbs.} = \cdot671957$$

(3) 1 lb. per cubic foot $= \frac{61027}{1728}$ lb. per cubic mètre
$$= \frac{61027}{2\cdot2046 \times 1728} \text{ kilogr.} = \frac{35\cdot217}{2\cdot2406} = 1\cdot6019$$

(4) 1 kilogr. per sq. mèt. $= \frac{1}{10\cdot7643}$ kilogr. per sq. ft
$$= \frac{2\cdot20462}{10\cdot7643} \text{ lbs.} = \cdot20481$$

(5) 1 cu. mèt. per kilogr. $= \frac{}{15432}$ cu. mèt. per grain
$$= \frac{61027}{15432} \text{ cu. in.} = 3\cdot9546$$

(6) Here we are to find how many circular areas each 1 inch in diameter are equal to 144 sq. inches; hence,

$$144) \underline{\cdot78539,} \text{ area of a circular inch}$$
$$\cdot005454$$

(7) 445 lbs. measure 1728 cu. in., ∴ 1 lb. = 1728÷445 = 3·883.

(8) 240*d.* : 7*d.* :: 100 : $\frac{35}{12}$ = 2·9167

(9) 27·7274 cu. in. weigh 1 lb., ∴ weight of 1728 cu. in. = 1728÷27·7274 = 62·321 lbs.
Also, 1 lb. measures 27·7274 cu. in., ∴ measure of 2240 lb. in cubic feet = $27\cdot7274 \times \frac{35}{27} = 35\cdot943$ cu. ft.

(10) $\frac{277\cdot274}{35 \times 1728}$ ton $= \frac{277\cdot274 \times 2240}{35 \times 1728}$ lbs. $= \frac{277\cdot274}{27}$
= 10·2694 lb.

(11) 1 ton per cu. yd. $= \frac{1}{1728 \times 27}$ ton per cu. inch

$= \frac{2240 \times 16}{1728 \times 27}$ oz. $= \frac{560}{729} = \cdot 76818$

(12) 1 grain per cu. inch = 1728 grains per cu. foot $= \frac{1728}{7000}$ lb. av. = ·246857.

AVERAGES AND PERCENTAGES.

(Page 131.)

2. (2625 + 9000 + 4750 + 27000 + 16500 + 3750 pence) + 6625 = 9·604*d.* average price;
6625 ÷ 6 = 1104, average sale;
63625*d.* ÷ 6 = 10604*d.* = £44 3*s.* 8*d.*, the average cash business.

3. To find the increase or decrease per cent. for each town, divide 100 times the absolute increase or decrease by the population of 1861. Thus, for the town A, there is an absolute decrease of 2127; hence 212700 ÷ 35134 = 6·054.

To find the total increase or decrease per cent. we have the whole population of 1861 = 814283, and that of 1871 = 835404, showing an increase of 21121 on 814283 = $\frac{2112100}{814283}$ per cent. = 2·594.

4. To find the mean of the percentages, divide the sums of each grade by 6. The gross percentages are found as follows:—

Good.		Fair.	
14 × 14·29 =	200·06	14 × 64·28 =	899·92
39 × 23·07 =	899·73	39 × 64·10 =	2499·90
22 × 22·73 =	500·06	22 × 68·18 =	1499·96
11 × 9·09 =	99·99	11 × 72·72 =	799·92
55 × 30·90 =	1699·50	55 × 65·45 =	3599·75
34 × 29·41 =	999·94	34 × 58·82 =	1999·88
175	) 4399·28	175	) 11299·33
	25·14		64·5

5. Sum of the offsets $+11 = 29{\cdot}2$ yds. $\div 11 = 2{\cdot}655$ yds. the mean distance; $\therefore$ 100 yds. $\times 2{\cdot}655 = 265{\cdot}5$ sq. yds. the approximate area.

6. (1) $\frac{9}{240}$ of 100 $= 3\frac{3}{4}$. (2) $\frac{7}{240}$ of 100 $= 2\frac{11}{12}$.
(3) $\frac{6}{240}$ of 100 $= 2\frac{1}{2}$. (4) $\frac{18}{240}$ of 100 $= 7\frac{1}{2}$.
(5) $\frac{80}{240}$ of 100 $= 33\frac{1}{3}$. (6) $\frac{90}{240}$ of 100 $= 37\frac{1}{2}$.
(7) $\frac{200}{240}$ of 100 $= 83\frac{1}{3}$. (8) $\frac{15}{960}$ of 100 $= 1\frac{9}{16}$.

7. France.

$59{\cdot}34 \times 11{\cdot}5 = 682{\cdot}41$
$112{\cdot}20 \times 9{\cdot}2 = 1032{\cdot}24$
$23{\cdot}39 \times 18{\cdot}4 = 430{\cdot}376$
$79{\cdot}54 \times 14{\cdot}0 = 1113{\cdot}56$
$47{\cdot}89 \times 13{\cdot}7 = 656{\cdot}093$
$93{\cdot}75 \times 8{\cdot}0 = 750{\cdot}0$
$416{\cdot}11$) $4664{\cdot}679$

In France 11·21 per cent.

Note.—The numbers for Italy and India, being treated as in the above calculation for France, will give the subjoined results.

Italy.	India.
693·55	1751·6
932·512	1648·65
305·04	292·243
596·85	758·784
950·214	650·96
134·1	621·027
266·74) 3612·266	461·81) 5723·264
In Italy 13·54 per cent.	In India 12·39 per cent.

The percentage altogether is found as follows :—

France,	416·11	4664·679
Italy,	266·74	3612·266
India,	461·81	5723·264
	1144·66	) 14000·209

Altogether 12·23 per cent.

8. As 65 per cent. of the army are at home, 35 per cent. are abroad; and we have the following proportions :—

	Artillery.	Cavalry.	Infantry.
At home,	$65 \times 10 = 650$;	$65 \times 15 = 975$;	$65 \times 75 = 4875$;
Abroad,	$35 \times 12 = 420$;	$35 \times 18 = 630$;	$35 \times 70 = 2450$;
	1070	1605	7325

Hence the percentages abroad are as follows :—

$(420 \times 100) \div 1070 = 39{\cdot}25$ per cent. of the artillery,
$(630 \times 100) \div 1605 = 39{\cdot}25$ „ „ cavalry,
$(2450 \times 100) \div 7325 = 33{\cdot}44$ „ „ infantry

INTEREST, &c.

(Page 139.)

1. (1) $£3{\cdot}75 \times 3\frac{1}{2} = \frac{7}{8}$ of $£15 = £13$ 2*s.* 6*d.*

(2) $£51{\cdot}91 \times 16\frac{2}{3} = £5191 \div 6 = £865$ 3*s.* 4*d.*

(3) $£10 \times 4\frac{1}{2} \times 5\frac{1}{2} = £45 \times 11 \div 2 = £247$ 10*s.*

(4) $5\frac{1}{4} + \frac{5\frac{1}{4} \times 198}{365} = £5$ 5*s.* $+ £2$ 16*s.* $11\frac{1}{2}d.$
$= £8$ 1*s.* $11\frac{1}{10}d$

(5) 40) £4728
Int. for 1 yr. £118 · 4
$\frac{219}{365} = \frac{3}{5} =$ 1 70 18 $4\frac{3}{4}$
£307 6 $4\frac{3}{4}$

(6) $£598{\cdot}74 \times 3\frac{11}{12} \times 3\frac{48}{74}$
or, $\frac{299{\cdot}37 \times 47 \times 89}{146}$
$£1252264{\cdot}71 \div 146$
$= £8577$ 3*s.* $1\frac{1}{4}d.$

2. (1) $£100 \times 100 \div 3\frac{1}{2} = £2857$ 2*s.* $10\frac{1}{4}d.$

(2) $£740 \div (£37 \times 3\frac{1}{2}) = 20 \div 3\frac{1}{2} = 5\frac{5}{7}$.

(3) $£1000 \div (£50 \times 4\frac{1}{2}) = 40 \div 9 = 4$ yrs. 162 da.

(4) $£7000 \times 100 \div (3\frac{1}{2} \times 5\frac{1}{4}) = £800000 \div 21$
$= £38095$ 4*s.* 9*d.*

(5) £10000 × 100 ÷ (7 × 1⅔) = £600000 ÷ 7
= £85714 5*s.* 9*d.*

(6) He obtained £492½ nett, for which he paid £525 at the end of 6 months; thus the actual half-year's interest was £32½; and therefore the rate = (65 × 100) ·492½ = 13⅛ per cent. per annum, nearly.

(7) *See* Answer in Text-Book.

(8) For every £100 lent £150 was received in 6 years. This was obtaining £50 as 6 years' interest on £100; ∴ 50 ÷ 6 = 8⅓ per cent. per annum.

3. (1) 89 : 100 :: £3 : £3 7*s.* 5*d.* per £100 per ann.

(2) $\frac{1375}{66}$ cents @ £2½ = £$\frac{6875}{1125}$ = £61 7*s.* 8*d.*

(3) $\frac{154\frac{1}{2}}{3\frac{1}{2}} = \frac{309}{7}$ cents = £30900 ÷ 7 = £4414 5*s.* 9*d.*

(4) £92 has produced £15 − £8, or £7, in 5 years,
= £1⅖ a year; hence,
92 : 100 :: 1·4 : 1$\frac{2}{3}$, or £1 10*s.* 5*d.* per £100.

(5) *See* Answer in Text-Book.

(6) £92 produces £3 a year; the investment of £85 produces $\frac{85 \times 5}{100}$ = £4¼ a year; and the increase of £1¼ a year will recover the loss of £7 in
7 ÷ 1¼ = 28 ÷ 5 = 5 yrs. 219 days.

4. (2)

		£927
2½ per cent.	= $\frac{1}{40}$ =	23·175
2 per cent.	= $\frac{1}{50}$ =	18·54
		968·715
	$\frac{1}{40}$ =	24·218
	$\frac{1}{50}$ =	19·374
		1012·307
	$\frac{1}{40}$ =	25·308
	$\frac{1}{50}$ =	20·246
		£1057·861 − £927

= £130·861 = £130 17*s.* 2½*d.*

(3)
10 p. c. $= \frac{1}{10} =$

£3750	
375	
4125	
412·5	
4537·5	
453·75	
4991·25	
499·125	
5490·375	

5490·375	
549·038	
6039·413	
603·941	
Amount	6643·354
	3750
Interest	2893·354

$= £2893\ 7s.\ 1d.$

(4) As the original principal $\times 1\frac{1}{10}$ successively would give the amounts for each year, that process is reversed by dividing by $1\frac{1}{10}$ or 1·1

1·1)£3750
3409·09091
3099·17355
2817·43050
2561·30046
2328·45496
£2116·77724
$= £2116\ 15s.\ 7d.$

(5) The reciprocal of $1\frac{1}{20}$ is $\frac{20}{21}$; therefore we may subtract $\frac{1}{21}$ successively.

21)£2279·17604
108·53219
2170·64385
103·36399
2067·27986
98·44190
1968·83796
93·75419
£1875·08377
$= £1875\ 1s.\ 8d.$

(6) The reciprocal of $1\frac{3}{20}$ is $\frac{20}{23}$.

£10000	
20	
23) 200000	
8695·65217	
20	
23) 173913·04340	
7561·43667	

7561·43667	
20	
23) 151228·73340	
6575·16232	
20	
23) 131503·24640	
£5717·53289	

$= £5717\ 10s.\ 8d.$

5. (1) 3408766 ÷ 2943578 = 1·158035, the ratio for 10 years; the square of this is 1·34104, the ratio for 20 years; hence, 3408766 × 1·34104 = 4571292.

(2) 967385÷1081954 = ·894109, the square of which is ·79943; hence, 967385 × ·79943 = 773·357.

(3) 42−27 = 15, the increase per thousand, = 1½ per cent.

	35743	37375·682
1 per cent. =	357·43	373·757
½ per cent. =	178·715	186·878
	36279·145	37936·317
	362·791	379·363
	181·396	189·682
	36823·332	38505·362
	368·233	35743
	184·117	2762, the increase
	37375·682	

PROFIT, ESTIMATES, PARTNERSHIP, &c.

(Page 146.)

'1. A, £168 for 5 mo. = £ 840 for 1 mo.
B, 70 „ 8 „ = 560 „
C, 400 „ 7 „ = 2800 „
4200

∴ A's share, $\frac{840}{4200} = \frac{1}{5}$ of £90 = £18,
B's „ $\frac{560}{4200} = \frac{2}{15}$ „ = 12,
C's „ $\frac{2800}{4200} = \frac{2}{3}$ „ = 60.

2. A, 56 for 90 days = 5040 for 1 day,
16 „ 60 „ = 960 „
6000;
B, 18 for 10 days = 180 for 1 day,
5 „ 12 „ = 60 „
240;
C, 36 for 60 days = 2160 for 1 day;
6000+240+2160 = 8400;
∴ B shall pay $\frac{240}{8400}$ of £5 = £$\frac{1}{7}$.
C „ „ $\frac{2160}{8400}$ of £5 = £1$\frac{2}{7}$.

3. I obtain an alloy consisting of nickel = $\frac{11}{23}$ of 10 lbs., lead = $\frac{5}{23}$ of 10 lbs., and tin = $\frac{7}{23}$ of 10 lbs.

But I require the lead to be $\frac{17}{19}$ of the nickel = $\frac{17}{19}$ of $\frac{11}{23}$ of 10 lbs., and the tin to be $\frac{11}{19}$ of the nickel = $\frac{11}{19}$ of $\frac{11}{23}$ of 10 lbs.; hence I must add ($\frac{17}{19}$ of $\frac{11}{23}-\frac{5}{23}$) of 10 lbs. = $\frac{40}{19}$ lbs. lead; and ($\frac{11}{19}$ of $\frac{11}{23}-\frac{7}{23}$) of 10 lbs. = $\frac{3180}{437}$ lb. tin.

4. 1st alloy contains nickel = $\frac{7}{16}$ of $\frac{5}{2}$ ton = 70 lbs. × 35 = 2450 lbs., tin = $\frac{5}{16}$ of $\frac{5}{2}$ ton = 70 lbs. × 25 = 1750 lbs., and lead = $\frac{4}{16}$ of $\frac{5}{2}$ ton = 70 lbs. × 20 = 1400 lbs.

2nd alloy contains nickel = $\frac{17}{47}$ of $\frac{47}{20}$ ton = 112 lbs. × 17 = 1904 lbs., tin = $\frac{29}{47}$ of $\frac{47}{20}$ ton = 2240 lbs., and lead = $\frac{18}{47}$ of $\frac{47}{20}$ ton = 1120 lbs.

The 5 tons consist of 2450+1904 = 4354 lbs. nickel,
1750+2240 = 3990 lbs. tin,
and 336+1400+1120 = 2856 lbs. lead;

Returns: 4354 lbs. nickel @ 1*s.* .....4354*s.*
3990 lbs. tin @ 5*d.* .....1662½*s.*
2856 lbs. lead @ 2*d.* ..... 476 *s.*
6492½*s.*

Outlay: 5600 lbs. alloy @ 5*d.* = 2333⅓*s.*
5264 lbs. alloy @ 5½*d.* = 2412⅔*s.*
336 lbs. lead @ 1½*d.* = 42 *s.*
4788
Profit = 1704½*s.*

4788 : 1704½ :: 100 : 170450÷4788
= 12175÷342 = 35·6 per cent.

5—7. *See* Answers in Text-Book.

8. By the Chain-Rule :—

? imp. gallons	= 157 wine gallons
if 1 wine gallon	= 231 cubic inches
1 cubic inch	= ·00001 6386 cu. mèt.
·001 cu. mèt.	= 1 litre
4·54347 litres	= 1 imp. gallon

$$\frac{157 \times 231 \times 16386}{4543470} = \frac{594271062}{4543470} = 130{\cdot}8 \text{ imp. gal.}$$

9. By the Chain-Rule :—

? roubles	= 1 Russian lb.
if 2442 Russ. lbs.	= 1000 kilogr.
45359 kilogr.	= 100000 Eng. lbs.
110231 Eng. lbs.	= 1000 centners
1 centner	= 375 thalers
7 thalers	= £1
£10	= 64 roubles

$$\frac{3750 \text{ thous.-millions} \times 32}{1221 \times 45359 \times 110231 \times 7} = \frac{120 \text{ millions}}{42734726} = 2{\cdot}80802 \text{ roub.}$$

10. By the Chain-Rule :—

? £	= 40·467 ares
if 1 are	= 100 sq. mèt.
10925 sq. mèt.	= 1750 roubles
1 rouble	= 3·95 francs
25·25 francs	= £1

$$\frac{40467 \times 175 \times 395}{10925 \times 2525} = \frac{22378251}{220685} = £101 \text{ 8}s. \text{ 1}d.$$

11. 1760×36 inches in 3600 seconds = 17·6 in. per second ; $17{\cdot}6 \div 39{\cdot}37 = {\cdot}447$ mètre per second.

12. 90° = 5400′ from pole to equator, = 12 in. × 6076½ × 5400 = 393757200; the 10-millionth of which = 39·37572 inches.

SPECIFIC GRAVITY.

(Page 152.)

1. $\frac{3}{8}\times\frac{10}{27}+\frac{3}{8}\times\frac{10}{17}=\frac{3}{8}+\frac{4}{17}=\frac{79}{153}$; ∴ the spec. gr. of the concrete = 153÷70 = 2·18571; hence

2185·71 oz.÷16 oz. = 136·61 lbs. *Ans.*

2. 2·1857 × 1·2 = 2·62. *Ans.*

3. Wt. of gold : wt. of quartz as $\frac{1}{2\cdot6}-\frac{1}{7}$: $\frac{1}{7}-\frac{1}{19}$, or as $\frac{11}{13}$: $\frac{6}{19}$, or as 209 : 78; hence

209÷(209+78) = ·7282 oz. gold.

4. ·7282 oz. at £3 17*s.* 10½*d.*

		3
		2·1846
15*s.*	¼	·54615
2*s.* 6*d.*	⅛	·09102
3*d.*	1/80	·00910
1½*d.*	1/160	·00455
		£2·8354

= £2 16*s.* 8½*d.*

5. Bulk of gold : bulk of quartz as 7−2·6 : 19−7, or as 11 : 30; hence 11÷(11+30) = 11/41 of 1 cu. in. = ·2683 of a cu. in.

6. $2\cdot4\times\frac{2-1}{2}=1\cdot2$

7. 13·5÷(1+31/40) = 540÷71 = 7·6056.

8. 3½ lbs. of water, at 10 lbs. per gallon, occupy 7/20 of the gallon; ∴ the material of which the sand is composed occupies 13/20 of the gallon; and as this weighs 16½ lbs. while 13/20 of a gallon of water weighs 6½ lbs., the sp. gr. required = 16½÷6½ = 2·54.

9. Let W denote the actual weight, S the specific gravity;

$$W-\frac{W}{S}=1{\cdot}53, \text{ and } W-\frac{{\cdot}85W}{S}=1{\cdot}73;$$

$$\therefore \left(1-\frac{{\cdot}85}{S}\right)\div\left(1-\frac{1}{S}\right)=173\div153; \text{ whence } S=2{\cdot}1475$$

Ans.

$$W=1{\cdot}73+\left(1-\frac{{\cdot}85}{2{\cdot}1475}\right)=\frac{1{\cdot}73\times859}{519}=\frac{859}{300}=2{\cdot}86\tfrac{1}{3} \text{ oz.}$$

Ans.

Also, as 1728 cu. in. weigh 2147·5 oz., the required bulk

$$=1728\times\left(\tfrac{859}{300}\div2147\tfrac{1}{2}\right)=1728\times\tfrac{1}{1250}$$
$$=6912\div3000=2{\cdot}304 \text{ cu. in.} \quad \textit{Ans.}$$

10. Instead of 20 and 15 we may use 4 and 3, which have the same ratio; hence,

$$4\times\tfrac{100}{847}={\cdot}47225$$
$$\tfrac{3}{7}\times\tfrac{100}{1115}={\cdot}26901$$
$${\cdot}74126$$
$${\cdot}1059$$
$$\tfrac{10}{95}={\cdot}10526$$
$${\cdot}1059 : {\cdot}00064 :: 100 : {\cdot}6 \text{ per cent. contraction.}$$

SQUARE AND CUBE ROOTS.

(Page 170.)

5. $\sqrt{(320\times4\times10\times121\times9)}=\sqrt{(2\times1600\times4\times121\times9)}$
$=2640\sqrt{2}=1{\cdot}414213\times2640=3733{\cdot}522$ ft.

6. $\sqrt{3232804}$ sq. po. = 1798 lineal poles = 29667 ft.

7. $\sqrt{(\frac{5}{8}\times\frac{5}{8})}=\frac{5}{4}=1\frac{1}{4}$ inch square.

8. Since iron is $\frac{2}{5}$ as heavy as gold, the end of the bar of gold will be $\frac{2}{5}$, or ·4, of a square inch; hence $\sqrt{{\cdot}4}$ = ·6324555 in.

9. $\sqrt{(162 \times 1058)} = \sqrt{(2 \times 81 \times 2 \times 529)} = 2 \times 9 \times 23 = 414.$

10. Here we are to find a mean proportional between a number and 7 times that number; the square root of 7 is 2·6457513, which $\times$ 3·3166248 gives 8·7749644.

13. $\sqrt[3]{(2 \times 12 \times 1029 \times 3)} = \sqrt[3]{(8 \times 27 \times 343)} = 2 \times 3 \times 7 = 42$ ft. $= 14$ yds.

14. $\sqrt[3]{(69 \times 3{\cdot}9 \times {\cdot}7)} = \sqrt[3]{188{\cdot}37} = 5{\cdot}7324$ in.

15. $(1 + 8\frac{3}{4})$ cu. ft. $= 4 \div 35 = {\cdot}114285714$ cu. ft., the cube root of which is ·4852855 of a foot. *Ans.*

$\sqrt[3]{8{\cdot}75}$ cu. ft. $= 2{\cdot}060643$ ft. *Ans.*

MACHINES, WORK, AND MOTION.

(Page 181.)

1. $(160 + 112)$ lbs. raised 20 times through 100 feet $= 272 \times 2000 = 544000$ foot-pounds.

2. The mean height to which the water has to be raised is 175 feet;

350×18 cu. ft. of water, weighing $350 \times 18 \times 62\frac{1}{2}$ lbs., to be raised through 175 feet;

$350 \times 9 \times 125 \times 175 = 68906250$ foot-pounds.

3. Work done upon 1 inch of each piston in 1 stroke $= 42 \times 10$.

Work upon a whole piston in 1 stroke $= 42 \times 10 \times 63{\cdot}6$;

do. in 25 strokes $= 42 \times 10 \times 63{\cdot}6 \times 144 \times 25$; therefore, the I.H.P. $= (42 \times 10 \times 63{\cdot}6 \times 144 \times 50) \div 33000 = 5828\frac{4}{55}$.—*See* Answer in Text-Book.

4. 20000 gal. water, weighing 10 lbs. per gallon = 200000 lbs., raised through 660 feet per minute;

$\therefore$ the effective H.P. $= (200000 \times 660) \div 33000$
$= 4000$;

$\therefore$ the efficiency $= 4000 \div 5828 = \cdot 686$.

5. $\frac{30 \times 2240 \times \frac{1}{2} \times 12}{4 \times 33000} = \frac{168}{55} = 3 \cdot 055$ H.P. done by each,
$3 \cdot 055 \div \cdot 6 = 5 \cdot 09$ H.P. applied to each.

6. $(5 \times 2240 \times 5 \times 25 \times 60) \div 33000 = 28000 \div 11$
$= 2545$ H.P.

7. 15 lbs. per square inch $= 144 \times 15$ lbs. per square foot on every side;

$\therefore 10 \times 144 \times 15 = 21600$ foot-pounds.

8. Height due to velocity $= \frac{V^2}{2g} = \frac{1440000}{64 \cdot 4} = 22360$ ft.;
$\frac{1}{2}(22360 \times 600) = 6708000$ foot-pounds.

9. $6708000 \div 33000 = 203 \cdot 3$ H. P.

10. Velocity due to the height $s = \sqrt{2gs} = \sqrt{(5280 \times 64 \cdot 4)}$
$= 8\sqrt{(33 \times 161)} = 72 \cdot 89 \times 8 = 583 \cdot 1$ ft., the muzzle velocity per second.

Work done by the shot $= (530 \div 7000)$ lbs. $\times 5280$;

$\therefore$ work done by the powder $= \frac{530 \times 5280}{7000} \div \frac{7}{10}$
$= (53 \times 528) \div 49 = 571$ ft.-pounds.

11. $(42 \times 5280) \div 3600 = 61{\cdot}6$ ft. per second.

Height due to velocity $= \frac{V^2}{2g} = \frac{61{\cdot}6^2}{64{\cdot}4} = 58{\cdot}92$ ft.

$170 \times 58{\cdot}92 = 10016$ foot-tons.

12. $(5 \times 5280) \div 3600 = 7\frac{1}{3}$ ft. per second.

Height due to velocity $= \frac{V^2}{2g} = \frac{(7\frac{1}{3})^2}{64{\cdot}4} = \frac{1210}{1449}$ ft.

$$10000 \times \frac{1210}{1449} = 8350 \text{ foot-tons.}$$

13. 5 kilom. per hour $= \frac{25}{18}$ mèt. per sec.

Height due to velocity $= \frac{625}{324 \times 9{\cdot}815 \times 2}$ mèt. per second.

Volume of water passing per second $= 1000 \times 10 \times \frac{25}{18}$ cu. mèt.

Hence $\frac{625 \times 10000 \times 25}{324 \times 19{\cdot}63 \times 18}$ cub. mèt. per sec.

$= \frac{625000000}{45793}$ cub. decimèt. or kilograms of water

= the no. of kilogrammètres of work per sec.;

which $\div 75 = 250000 \div 13738$

$= 18197$ H.P.

14—24. *See* Answers in Text-Book.

25. $\sqrt{(64{\cdot}4 \times 22000)} = \sqrt{1416800} = 1190$ ft. per sec.

26. This is equivalent to lowering one half of the quantity of water 15 feet; and as sea-water weighs 64 lbs. or $\frac{1}{35}$ of a ton per cubic foot, the work is

$$\frac{6 \times 4840 \times 9 \times 15 \times 30}{35} = 3360343 \text{ foot-tons.}$$

27. Height due to velocity $= 48^2 \div 64{\cdot}4 = 24^2 \div 16{\cdot}1 = 35{\cdot}8$ feet; hence the total height proportional to the work is 135·8 ft. The delivery is $\frac{48 \times 4\frac{1}{2}}{144} = \frac{3}{2}$ cub. ft. per second $= \frac{3}{2}$ of $62\frac{1}{2}$ lbs. or 93·75 lbs. per second. Therefore the effective H.P.

$$= \frac{\cdot 55 \times 60}{33000} \times 135{\cdot}8 \times 93{\cdot}75 = 12{\cdot}73.$$

28. Six miles an hour $= 8{\cdot}8$ feet per second; and the height due to this $= 8{\cdot}8^2 \div 64{\cdot}4 = 4{\cdot}4^2 \div 16{\cdot}1 = 1{\cdot}2$ ft.

29. Work may be lost both in getting up velocity and bringing to rest. The height due to a velocity of 500 feet per minute, or $8\frac{1}{3}$ feet per second, is $(8\frac{1}{3})^2 \div 64{\cdot}4 = \frac{3125}{2898}$ ft. and $112 \times 12 \times \frac{3125}{2898}$ lb., or $\frac{32 \times 3125}{69}$ lb. may be lost 4 times at each double stroke, or 76 times a minute. The H.P. therefore which may possibly be lost by this reciprocation is

$$\frac{32 \times 3125 \times 76}{33000 \times 69} = \frac{7600}{33 \times 69} = 3{\cdot}3.$$

30. The pressure of the atmosphere being 15 lbs. to the square inch, we have $70 - 15$, or 55 lb. to the square inch, equal to a head of water of $\frac{55 \times 144}{62\frac{1}{2}}$ or 127 feet. The velocity therefore $= \sqrt{127 \times 64{\cdot}4} = 90{\cdot}4$ ft. per second.

31. Here the upward pressure is uniform, and the work done per minute is 33000×100 foot-pounds; $\therefore$ the lift (or speed) per minute $= \frac{3300000}{1400 \times 2240} = 1{\cdot}05$ ft.

32. Height due to velocity $= 50^2 \div 64{\cdot}4 = 38{\cdot}82$ feet, and the work of the hammer is $30 \times 10 \times 38{\cdot}82 = 11646$ foot-tons per minute, or 791 H.P.; one-fourth of which $= 198$ H.P. is spent in shaking the foundation.

SIMILAR FIGURES.

(Page 189.)

1. $\sqrt[3]{4} = 1{\cdot}5874$; this $\times 11\frac{1}{2}$ and 21 gives 18·255 in. and 33·335 in.

2. $(11\frac{1}{2})^2 \times 21 : (17\frac{1}{4})^2 \times 31\frac{1}{2} :: 9$ galls.
or $23 \times 23 : 23 \times 23 \times 9 \times \frac{3}{8} :: 9 : 24\frac{3}{8} = 30\frac{3}{8}$ galls.

3. $21 : 31\frac{1}{2} :: 1^3 : 1{\cdot}5$; and $\sqrt[3]{1{\cdot}5} = 1{\cdot}147$ in.

4. $200^3 : 300^3 :: 1800$ tons $: 1800 \times 3\frac{3}{8} = 6075$ tons

5. $200^2 : 300^2 :: 14$ cwts. $: 14 \times 1{\cdot}5^2 = 31\frac{1}{2}$ cwts.

6. $1 \times \sqrt{2 \times 2 \times 2} = 2\sqrt{2} = 2{\cdot}8284$ galls.

MEASUREMENT OF LINES.

(Page 196.)

1. $\sqrt{12^2+4^2} = \sqrt{160} = 12{\cdot}649$ ft.
$\sqrt{12^2+3^2} = \sqrt{153} = 12{\cdot}369$ ft.
$\sqrt{4^2+3^2} = 5$ ft. $\sqrt{12^2+4^2+3^2} = 13$ ft.

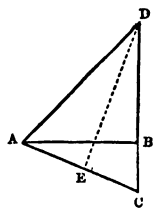
2. $\sqrt{25^2-7^2} = \sqrt{32 \times 18} = \sqrt{64 \times 9} = 8 \times 3 = 24$ ft.

3. Area of the square $= 512 \times 256 = 256^2 \times 2$;
$\therefore$ side $= 256\sqrt{2} = 1{\cdot}41421356 \times 256 = 362{\cdot}03867$ ft.

4. Edge $= 13{\cdot}69 \div \sqrt{3} = 13{\cdot}69 \times \frac{1}{3}\sqrt{3}$; $\therefore$ the volume $= 13{\cdot}69^3 \times \frac{1}{3}\sqrt{3} = 2565{\cdot}7264 \times {\cdot}19245 = 493{\cdot}774$ cub. mèt.

5. The sides are $\frac{1}{2}(23+7)$ and $\frac{1}{2}(23-7)$, or 15 and 8 yds.; $\therefore \sqrt{15^2+8^2} = 17$ yds.

6. Let AB = 100 yards; draw BC = 40 yards perpendicular to AB; join AC, and produce CB to D so that the angle CAD shall be equal to the angle ACD; bisect AC by the perpendicular DE; then ADB is the triangle proposed.



Now $AC^2 = AB^2+BC^2 = 11600$, $\therefore EC^2 = 2900$; and the triangles ABC and CED being equiangular are similar;

$\therefore BC^2 : CA^2 :: EC^2 : CD^2$, that is, 1600 : 11600 :: 2900 : CD^2; whence $CD^2 = AD^2 = 29^2 \times 5^2$, $\therefore$ AD = 145 yds., and CD − CB = DB = 105 yds.

Otherwise, by Algebra: Let ABD be the triangle, AB = 100, AD = $x+20$, $\therefore$ DB = $x-20$; then $AD^2 - DB^2 = 80x = AB^2 = 10000$; whence $x = 125$, &c.

7. 180° = $\frac{355}{113}$ times radius; $\therefore (180° \times 1\frac{1}{2}) \div \frac{355}{113}$ = $113 \times 54 \div 71 = 85°\ 56'\ 37{\cdot}2''$

8. 39·37 in. $\times \frac{355}{113} = 123{\cdot}68$ in.

9. $\frac{54}{180}$ of $27 \times \frac{355}{113} = \frac{81}{2}$ of $\frac{71}{113} = 25{\cdot}447$ in.

10. $\frac{180}{72}$ of $30 \times \frac{113}{355} = 15 \times \frac{113}{71} = 23{\cdot}873$ in.

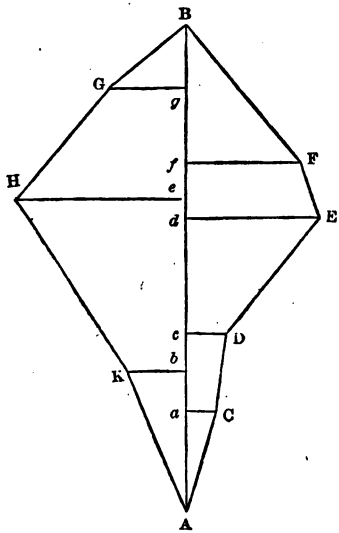
11. $41847662 \times \frac{355}{113} \div (360 \times 60) = \frac{6877774}{113} = 6086{\cdot}5$ ft.

12. When diam. = 1 inch, circumf. = $\frac{355}{113 \times {\cdot}3937}$ centimèt. = 7·97966, which ÷ 8 gives ·99745; should be 1; $\therefore$ ·00255 on 1 = ·255 on 100, or about $\frac{1}{4}$ per cent. in defect.

MEASUREMENT OF RECTILINEAL AREAS.

(Page 207.)

1. (1) $75 \times 300 = 22500$ sq. yds.
   (2) $\frac{1}{2}(350 \times 200) = 35000$ sq. yds.
   (3) $(150+120) \times 275 = 74250$ sq. ft.
   (4) $\frac{1}{2}(140+170) \times 90 = 13950$ sq. yds.



(5) We have A*a* 50, *ab* 20, *bc* 20, *cd* 60, *de* 10, *ef* 20, *fg* 40, *g*B 30; the offsets are *a*C 15, *c*D 20, *d*E 70, *f*F 60, *g*G 40, *e*H 90, *b*K 30. Hence the following areas:

A*a*C	375	B*g*G	600
*a*D	700	*g*H	3900
*c*E	2700	*e*K	5400
*d*F	1950	A*b*K	1050
B*f*F	2100		10950
	7825		7825

Whole area, 18775 sq. yds.

(6) *See* Answer in Text-Book.

2. (1) $\sqrt{(172^2+93^2)} = \sqrt{38233} = 195{\cdot}5326$ yds., diag.;
$172 \times 93 = 15996$ sq. yds., area.

(2) The given Nos. $= 5 \times 88$ and 3×88; hence the length and breadth are 4×88 and 3×88, and the product of these $= 12 \times 88 \times 88 = 92928$ sq. yds.

(3) $\sqrt{(26+10)} \times \sqrt{(26-10)} = 6 \times 4 = 24$ ft.

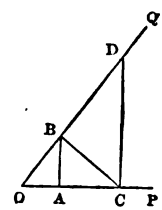
(4) $\sqrt{(3^2+4^2+12^2)} = \sqrt{169} = 13$ ft.

(5) Slope width of each side of the roof $= \sqrt{(15^2+10^2)}$ $= 18{\cdot}0278$ ft.; $\therefore$ quantity of slating $= 50 \times 18{\cdot}0278 \times 2$ $= 1802{\cdot}78$ sq. ft. $= 200{\cdot}31$ sq. yds.

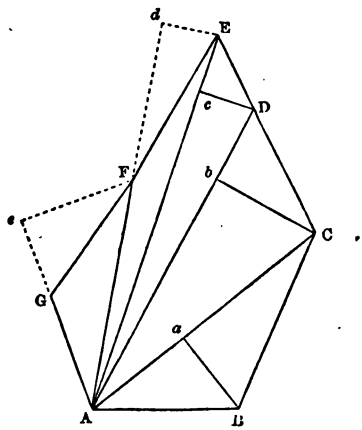
(6) Perpendicular length of the triangle forming either end $= \sqrt{10^2+10^2} = \sqrt{200}$. Each of the equal sides of that triangle $= \sqrt{200+10^2} = \sqrt{300}$; $\therefore$ breadth of each side of the roof $= \sqrt{300-10^2} = \sqrt{200} = 14{\cdot}14214$ ft. Twice the area of a side $= 14{\cdot}14214 \times (50+30)$, twice the area of an end $= 14{\cdot}14214 \times 20$; $\therefore$ whole area $= 14{\cdot}14214 \times 100$ $= 1414{\cdot}214$ sq. ft.

(7) Hypotenuse $= 17$; $\therefore$ $(15 \times 8) \div 17 = 7{\cdot}0588$ yds.

(8) OA $= 3$, AB $= 4$, $\therefore$ OB $= 5$;
OA : AB :: OB : BC $= 6\frac{2}{3}$ in.
OA : OB :: OB : OC $= 8\frac{1}{3}$ in.
OA : OC :: OB : OD $= 11\frac{2}{3}\frac{5}{5} = 13\frac{8}{15}$ in.



3. (1) The areas of the successive triangles A B C, A C D, &c., are 15, 24, 14, 8, 8; ∴ whole area = 69 sq. in.

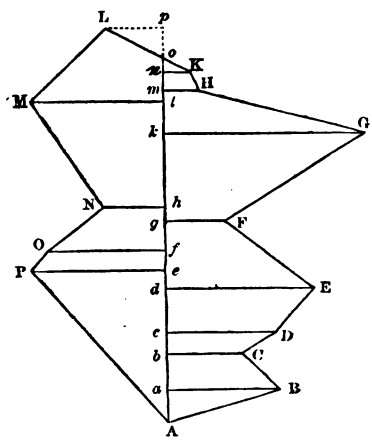


A a = $\sqrt{(5^2-3^2)}$ = 4; B C = $\sqrt{(3^2+6^2)}$ = 6·7082 in; A b = $\sqrt{14\times6}$ = 9·16515, ∴ D b = 2·83485; whence C D = $\sqrt{(16+8\cdot036375)}$ = 4·9027 in.; A c = $\sqrt{14\times10}$ = 11·83216, ∴ E c = 2·16784; whence D E = $\sqrt{(4+4\cdot69953)}$ = 2·9495 in.; A d = $\sqrt{16\times12}$ = $8\sqrt{3}$ = 13·856406; ∴ F d = 5·856406; whence E F = $\sqrt{(4+34\cdot29749)}$ = 6·1885 in.; A e = $\sqrt{12\times4}$ = $4\sqrt{3}$ = 6·928203, ∴ G e = 2·928203; whence F G = $\sqrt{(16+8\cdot57437)}$ = 4·9572 in.

(2) Radius × 4·5 = twice the area; ∴ 4·5 ÷ 4 = 1·125 sq. ft.

(3) $96-(4\times10+4\times6)$ = 32 sq. in.; ∴ $(32\times2)\div4$ = 16 in.

(4) Area = OAB + OBC+&c. = 25 + 84 + 104 + 170 + 132+88+40½+10½ = 654 sq. in.



(5) We have A*a* 10, *ab* 12, *bc* 8, *cd* 15, *de* 8, *ef* 7, *fg* 10, *gh* 5, *hk* 25, *kl* 10, *lm* 5, *mn* 7, *np* 13; the offsets are *a*B 40, *b*C 25, *c*D 38, *d*E 50, *g*F 15, *k*G 70, *m*H 10, *n*K 8, L*p* 20, M*m* 50, N*h* 25, O*f* 40, P*e* 50. The segments of *np*, viz. *no*, *op*, being in the ratio of *n*K to L*p*, that is, as 2 to 5, we have $no = \frac{2}{7}$ of $np = 3\frac{5}{7}$, and $op = \frac{5}{7}$ of $np = 9\frac{2}{7}$. Hence the following areas :—

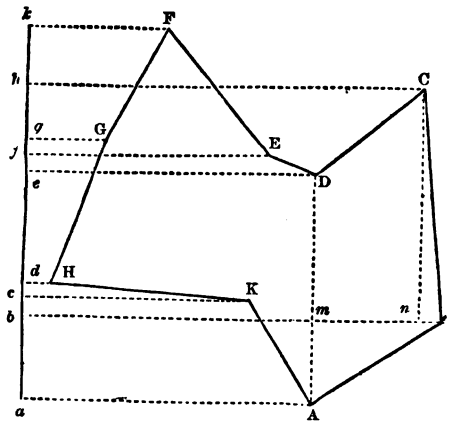
A*a*B	200
*a*C	390
*b*D	252
*c*E	660
*d*F	812½
*g*G	1275
*k*H	600
*m*K	63
*on*K	14⅙
	4267½

*p*M	875
*op*L	92⅙
L*om*M	782⅐
*l*N	1312½
*h*O	487½
*f*P	315
A*e*P	1325
	4222
	4267½
	8489½ *Ans.*

Note.—The fractions ⅙ and ⅐ amounting to 1, we have reckoned the first column as 1, and neglected ⅐ in the second,

(6) We have *ab* 30, *bc* 5, *cd* 5, *de* 40, *ef* 4, *fg* 6, *gh* 20, *hk* 20; the offsets are *a*A 100, *b*B 150, *c*K 80, *d*H 10, *e*D 100, *f*E 90, *g*G 30, *h*C 140, *k*F 50. Hence the following areas :—

kE = $\frac{1}{2}(50+90)\times 46$	=	3220
fD = $\frac{1}{2}(90+100)\times 4$	=	380
aD = 100×80	=	8000
AmB = $\frac{1}{2}(30\times 50)$	=	750
CnB = $\frac{1}{2}(80\times 10)$	=	400
mC = $\frac{1}{2}(50+80)\times 40$	=	2600
		15350



kG = $\frac{1}{2}(50+30)\times 40$	=	1600
gH = $\frac{1}{2}(30+10)\times 50$	=	1000
cH = $\frac{1}{2}(10+80)\times 5$	=	225
aK = $\frac{1}{2}(80+100)\times 35$	=	3150
deduct . . .		5975
Ans.		9375 sq. yds.

4. (1) (*a*) $\frac{1}{2}\times 6 = 3$, which is ·141592 in defect.

(*b*) $1{\cdot}1547005\times 3 = 3{\cdot}4641015$, which is ·322509 in excess.

(*c*) $\frac{1}{2}(3+3{\cdot}4641015) = 3{\cdot}232051$, which is ·090459 in excess.

(*d*) $\frac{2}{3}$ of $3+\frac{1}{3}$ of $3{\cdot}4641015 = 3{\cdot}1547005$, which is ·013108 in excess.

(2) *See* Answer in Text-Book.

(3) (*a*) $2{\cdot}5980762 \div 4 = {\cdot}64951905$; giving $-{\cdot}135879$.

(*b*) ${\cdot}57735025 \times \frac{1}{4} \times 6 = {\cdot}866025$; giving $+{\cdot}080627$.

(*c*) $\frac{1}{2}({\cdot}649519 + {\cdot}866025) = {\cdot}757772$; giving $-{\cdot}027626$.

(*d*) $\frac{1}{3}$ of ${\cdot}64951905 + \frac{2}{3}$ of ${\cdot}86602537 = {\cdot}793857$; giving $+{\cdot}008459$.

(4) (*a*) ${\cdot}258819^2 = {\cdot}0669873$; this $\times 11{\cdot}1961524$ gives ${\cdot}75$; error $= -{\cdot}035398$.

(*b*) ${\cdot}2679492^2 = {\cdot}0717968$; this $\times 11{\cdot}1961524$ gives ${\cdot}803848$; error $= +{\cdot}018450$.

(*c*) $\frac{1}{2}({\cdot}75 + {\cdot}803848) = {\cdot}776924$; error $= -{\cdot}008474$.

(*d*) $\frac{1}{3}$ of $\cdot75 + \frac{2}{3}$ of ${\cdot}803848 = {\cdot}785899$; error $= +{\cdot}000501$.

(5) Each of the four corners cut off from a square to form the octagon has each of its two equal sides $= 50\sqrt{2}$; $\therefore$ the side of the square $= 100(1 + \sqrt{2})$, and its area $= 10000(3 + 2\sqrt{2})$; $\therefore$ area of octagon $= 20000(1 + \sqrt{2})$ $= 2{\cdot}41421356 \times 20000 = 48284\frac{1}{4}$ sq. yds.; which $\div 484$ gives 99 sq. chains $368\frac{1}{4}$ yds. = 9 ac. 9 ch. $368\frac{1}{4}$ yds.

(6) The square of a side = (4840 sq. yds. $\times$ 500) $\div 9{\cdot}3656404 = 258391$; the sq. root of which is $508{\cdot}32$ yds.;
$\therefore$ $508{\cdot}32 \times 11 = 5591{\cdot}52$ yds.

MEASUREMENT OF CURVED AREAS

By Means of Offsets.

(Page 217.)

1.

$26 \times 1 =$	26
$37 \times 4 =$	148
$50 \times 2 =$	100
$65 \times 4 =$	260
$82 \times 1 =$	82
	616
$\frac{1}{3}$ interval	10
Area	6160 sq. in.

2.

$74 \times 1 =$	74
$63 \times 4 =$	252
$50 \times 2 =$	100
$35 \times 4 =$	140
$18 \times 1 =$	18
	584
$\frac{1}{3}$ interval	10
Area	5840 sq. in.

Note: 26 + 74 = 37 + 63 = &c. = 100, which × the base, 120, = 6160 + 5840 = 12000, the joint area.

3. The chord of the segment to radius 1 = $\sqrt{2}$, and the perpendicular from the centre to the chord = $\frac{1}{2}\sqrt{2}$; hence the area of the triangle of the sector = ·5, which subtracted from the area of the sector, $\frac{1}{4}\pi$ or ·7854, leaves ·2854, the true area of the segment.

But the question is intended for practice by actual drawing and measurement. If we divide the chord into four, the interval will be $\frac{1}{4}\sqrt{2}$ = ·35355, and the ordinates will be 0, 0·2283, 0·2929, 0·2283, 0; hence,

0 × 1 =	0
0·2283 × 4 =	0·9132
0·2929 × 2 =	0·5858
0·2283 × 4 =	0·9132
0 × 1 =	0
	2·4122
$\frac{1}{3}$ interval	·11785
Area	·28428

This is less than the true area by ·0011, or about $\frac{2}{3}$ per cent.

4.

Trap. Rule.	Simpson's Rule.
·05	·1 × 1 = ·1
·14	·14 × 4 = ·56
·173	·173 × 2 = ·346
·20	·20 × 4 = ·8
·224	·224 × 2 = ·448
·245	·245 × 4 = ·98
·265	·265 × 2 = ·53
·283	·283 × 4 = 1·132
·15	·3 × 1 = ·3
1·73	3) 5·196
	1·732

5. Omitting the alternate ordinates, we have

Trap. Rule.	Simpson's Rule.
·05	·1 × 1 = ·1
·173	·173 × 4 = ·692
·224	·224 × 2 = ·448
·265	·265 × 4 = 1·06
·15	·3 × 1 = ·3
·862	2·600
2	2
1·724	3) 5·200
	1·733 sq. ft.

Using the smaller number of ordinates we get the answer exact by Simpson's rule; this is because the ordinates (as usually occurs in practice) are approximate instead of exact, and there has been a compensation of errors. The error of the trapezoidal rule is increased from ·003 in defect to ·009 in defect.

6. Simpson's rule is not adapted to this. By the trapezoidal rule we get

$$0\cdot5+1\cdot26+1\cdot44+1\cdot59+1\cdot71+1\cdot82+1\cdot9+1 = 11\cdot22$$

centimèt.; which × 1·4 centimèt. gives 16·708 square centimètres = 2·59 English square inches.

AREAS OF CIRCLES.

(Page 220.)

1. $10^2 \times \cdot 785398 = 78 \cdot 5398$ sq. in.

2. $7^2 \times \cdot 785398 = 38 \cdot 4845$ sq. in.

3. Reciprocal of $\pi = 1 \div 3 \cdot 1416 = \cdot 31831$; which $\times 10$ gives the square of the radius; $\therefore \sqrt{3 \cdot 1831} = 1 \cdot 784$ in.

4. The area = the radius $\times \frac{1}{2}$ of $27 = (\frac{27}{2})^2 \div \pi$
$= \frac{729}{4} \times \cdot 31831 = 58 \cdot 012$ sq. in.

5. Half the circumf. $\times$ the radius = 1 sq. mi.;
$\therefore$ the circumf. $= 2\sqrt{\pi} = 1 \cdot 77245 \times 2 = 3 \cdot 5449$ mi.

6. If 2 = the diam. of the circle, and therefore = a side of the square, the area of the circle $= 2^2 \times \frac{1}{4}\pi$, and that of the square $= 2^2$; these areas then are as $\pi : 4$. Also, the perimeter of the circle $= 2\pi$, and that of the square $= 8$; and these again are as $\pi : 4$.

7. $(100^2 - 80^2) \times \cdot 785398 = 2827 \cdot 4328$ sq. yds.
$= 93$ po. $14 \cdot 18$ yds. $= 2$ ro. 13 po. $14 \cdot 18$ yds.

8. If d denote the internal diameter, we have $6^2 - d^2$
$= 9 \div \frac{1}{4}\pi = 36 \times \cdot 31831 = 11 \cdot 4592$;

$\therefore d = \sqrt{36 - 11 \cdot 4592} = \sqrt{24 \cdot 5408} = 4 \cdot 954$ in. internal diameter; $\therefore$ radius $= 2 \cdot 477$ in.

9. The square of the required circumf. $= (17^2 - 15^2)$
$= 32 \times 2$; $\therefore$ the circumf. $= 8$ in.

10. The area of a regular pentagon, in terms of the square on a side $= 1{\cdot}72048$; this $\times(2^2-1^2) = 5{\cdot}1614$ sq. in.

11. When the diameter $= 1$, the area $= {\cdot}785398$; and $\frac{101}{100}$ of $\frac{7}{9} = {\cdot}785555 =$ about ·0002 in excess.

SECTORS AND SEGMENTS OF CIRCLES.

(Page 223.)

1. $45° = \frac{1}{8}$ of the circumf.; $\therefore$ the sector $= \frac{1}{8}$ of $3^2\times\pi$
$= 1\frac{1}{8}$ of $3{\cdot}1416 = 3{\cdot}5343$ sq. ft.

2. The area of the circle = square of circumf. $\div 4\pi$; and as $\frac{59}{3600}$ of this $= 1$ sq. inch, we have the square of the circumf. $= \frac{1}{3}$ of 144π; $\therefore$ the circumf. $= 12\sqrt{{\cdot}628318}$
$= 9{\cdot}5119$.

3. The angle $= 10\div(\frac{1}{3600}$ of $3^2\times\pi)$
$= 3600\div(3^2\times\pi) = 400\times{\cdot}31831$
$= 127{\cdot}324$ deg. $= 127°\ 19'\ 26''$

4. Area of sector $= \frac{1}{4}$ of $4^2\times\pi = 4\pi = 12{\cdot}5664$
area of triangle $= \frac{1}{2}(4\times4) = \underline{8{\cdot}0}$
area of segment $4{\cdot}5664$ sq. in.

5. Area of sector $= \frac{1}{6}$ of $6^2\times\pi = 6\pi = 18{\cdot}84955$
area of the equilateral triangle $= 9\sqrt{3} = \underline{15{\cdot}58846}$
$3{\cdot}26109$ sq. in.

6. $\cdot809\times13 = 10{\cdot}517$ in. perp. from extremity of arc to radius.
Area of sector $= \frac{54}{360}$ of $13^2\times\pi = \frac{3}{20}$ of 169π
$= 79{\cdot}6394$
area of triangle $= \frac{1}{2}(10{\cdot}517\times13) = \underline{68{\cdot}3605}$
$11{\cdot}2789$ sq. in.

7. Square of radius $= \frac{1}{2}$ of $200^2 = 20000$
area of sector $= \frac{1}{4}$ of $20000\pi = 15708$
area of triangle $= \frac{1}{4}$ of $200^2 = 10000$
5708 sq. ft.

8. For a span of 240 ft. with a rise of 19·006 ft. we have
$DF = 120^2 \div 19{\cdot}006 = 757{\cdot}655$
$19{\cdot}006$
diam. $CF = 776{\cdot}661$
radius $= 388{\cdot}330$
$19{\cdot}006$
$OD = 369{\cdot}324$
area of triangle $= OD \times 120 = 44319$ sq. ft.
area of circle $= 388{\cdot}33^2 \times \pi = 150800 \times 3{\cdot}1416$,
$\frac{1}{10}$ of which $= 47375$ sq. ft., area of sector
44319
3056 sq. ft., area of segment.

Note.—CD = 19·006 = 120 (cosec 18° − cot. 18°).

THE ELLIPSE.

(Page 227.)

1. $3 \times 2 \times {\cdot}7854 = 4{\cdot}7124$ sq. in.

2. $2 \times 1 \times {\cdot}7854 = 1{\cdot}5708$ sq. mi.

3. $4\frac{1}{2} \times 3 \times {\cdot}7854 = 10{\cdot}603$ sq. ft.

4. Calling the axes a and b, the area of the ellipse $= ab \times {\cdot}7854$, and that of the parallelogram $= \frac{1}{2}ab$; $\therefore$ area of each segment $= \frac{1}{4}ab \times ({\cdot}7854 - {\cdot}5) = ab \times {\cdot}07135$.

5. Calling the external and internal diameters of the ring D and d, its area $= (D^2 - d^2) \times \frac{1}{4}\pi = (D+d)(D-d) \times \frac{1}{4}\pi$ = the area of an ellipse having $a = D+d$ and $b = D-d$.

6. $25 \div (8 \times {\cdot}7854) = 3{\cdot}979$ in.

7. Rad. $= \frac{1}{2}\sqrt{24 \times 36} = 6\sqrt{6} = 14{\cdot}697$ in.

8. Area of parallelogram $= 14 \times 3 = 42$; $\therefore$ area of ellipse $= {\cdot}785398 \times 42 = 32{\cdot}9867$ sq. in.

VOLUME.

(Page 232.)

1. $(1{\cdot}5 \times 3 \times 1728) \div 277{\cdot}274 = 28$ gall.

2. $9^2 \times {\cdot}785398 \times 10 = 636{\cdot}17238$ cu. ft.
$= 636$ cu. ft. 298 in.

3. $(8 \times 6 \times {\cdot}785398 \times 12 \times 1728) \div 277{\cdot}274$
$= 781728{\cdot}62 \div 277{\cdot}274 = 2819$ gal.

4. $13 \times 14 \times \frac{1}{3}$ of $18 = 1092$ cu. ft.

5. $9 \times 7 \times \frac{1}{3}$ of $17 = 51 \times 7 = 357$ cu. ft.

6. $D^2 \times \frac{1}{4}\pi \times 6 = 277{\cdot}274$ cu. in.
$\therefore D^2 = 277{\cdot}274 \times {\cdot}31831 \times \frac{2}{3}$; and $D = \sqrt{58{\cdot}8394}$
$= 7{\cdot}67$ in.

7. $D^3 \times \frac{1}{4}\pi \times D = 277{\cdot}274$; or $D^3 = 1109{\cdot}096 \times {\cdot}31831$;
$\therefore D = \sqrt[3]{353{\cdot}03635} = 7{\cdot}068$ in.

8. Content $= 3^2 \times \pi \times \frac{1}{3}$ of $5 = 15\pi = 47{\cdot}1239$ cu. in.
Weight of a cu. in. of mercury $= \dfrac{62\frac{1}{2} \text{ lbs.} \times 13\frac{1}{2}}{1728}$

$$= \frac{125}{256} \text{ lb.}$$

$\therefore 47{\cdot}1239 \times \frac{1000}{2048} = 47123{\cdot}9 \div 2048 = 23{\cdot}01$ lbs.

9. $(18^2 - 7^2) \times {\cdot}785398 \times \frac{144}{1728} = \frac{1}{12}(275 \times {\cdot}785398)$
$= 216 \div 12 = 18$ cu. ft.;
$18 \times 7{\cdot}1 \times 62\frac{1}{2}$ lbs. $= 7987$ lbs. $= 3$ t. 11 cwt. 35 lbs.

10. $5^2 \times \cdot 785398 \times 160 = 3141 \cdot 592$ cu. ft. of water = 196349 lbs. to be raised through a mean height of $80 + 40$, or 120 ft., making $(196349 \times 120) \div 2240 = 10519$ foot-tons.

11. *See* Answer in Text-Book.

12. Volume of water in cubic feet $= (\frac{7}{12})^2 \times \pi \times 720$, raised through 102 ft. per minute;

$$\text{hence } \frac{49}{144} \times \pi \times 720 \times 102 \times \frac{1000}{16 \times 33000} = \frac{49\pi \times 85}{88}$$
$$= 148 \cdot 69 \text{ H.P.}$$

VOLUME.

(Page 234.)

1. $62\frac{1}{2} : 2\frac{1}{2} :: 1728$ cu. in. : 69·12 cu. in. in the empty cylindric ungula;

$$\therefore \frac{69 \cdot 12 \times 2}{5^2 \times \cdot 7854} = \frac{138240}{19635} = 7 \cdot 04 \text{ in.}$$

2. Capacity of glass $= 3^2 \times \frac{1}{4}\pi \times \frac{1}{3}$ of $4 = 3\pi$;
$\therefore$ quantity of water $= \frac{3}{4}\pi = 4 \cdot 7124$ cu. in.;
then $\sqrt[3]{2} : \sqrt[3]{1}$ (or $\sqrt[3]{8} : \sqrt[3]{4}$) $:: 4$ in. : $\sqrt[3]{4 \times 2}$,
$= 3 \cdot 1748$ in. the height of water.

3. Volume of pyramid $= 80 \times 60 \times \frac{1}{3}$ of $50 = 80000$ cu. ft.
$50^3 : 30^3 :: 80000 : 17280$ cu. ft. cut off;
$\therefore$ $80000 - 17280 = 62720$ cu. ft. left.

4. $24 - 10 : 10 :: 60$ ft. : $42\frac{6}{7}$ ft., the height wanting to complete the pyramid.
Volume of pyramid $= 2^2 \times \frac{1}{4}\pi \times \frac{1}{3}$ of $102\frac{6}{7}$
$= 34 \cdot 28556\pi$;
Volume of upper pyramid $= (\frac{5}{8})^2 \times \frac{1}{4}\pi \times \frac{1}{3}$ of $42\frac{6}{7}$
$= 2 \cdot 48016\pi$;
$\therefore$ volume of frustum $= 31 \cdot 8054\pi = 99 \cdot 92$ cu. ft.

5. $80-30 : 30 :: 55$ ft. : 33 ft., height wanting to complete the pyramid. Area of base $= 10 \times 10 \times 4{\cdot}8284$; do. of top $= \frac{30}{8} \times \frac{30}{8} \times 4{\cdot}8284$.
$(100 \times \frac{88}{3} - \frac{225}{16} \times 11) \times 4{\cdot}8284 = \frac{133375}{48} \times 4{\cdot}8284$
$= 13416$ cu. ft.

6. $(4^2 \times 5 - 3^2 \times 3) \times \frac{1}{3}$ of $\cdot7854 = \cdot2618 \times 53 = 13{\cdot}875$ cu. ft.

7. $(12^2 \times \frac{12}{3} - 10^2 \times \frac{10}{3}) \times \cdot7854 = 190{\cdot}59$ cu. in. of brass.
Weight $= \frac{190{\cdot}59}{1728} \times \frac{8000}{16} = 55{\cdot}147$ lbs. $= 55$ lbs. 2·35 oz.

8. $(\frac{12}{3} - \frac{10}{3}) \times 8^2 \times \cdot7854 = 33{\cdot}5104$ cu. ft. $= 57905{\cdot}97$ cu. in., which $\div 277{\cdot}274$ gives 208·84 gal.

SPHERES, etc.

(Page 238.)

1. $7^3 \times \cdot523597 = 179{\cdot}594$ cu. in.

2. $277{\cdot}274 \div \cdot523597 = 529{\cdot}556$;
$\sqrt[3]{529{\cdot}556} = 8{\cdot}090$ in. diam.; $\therefore$ radius $= 4{\cdot}045$ in.

3. Diam. $= \sqrt[3]{\frac{6}{\pi}} = \sqrt[3]{6 \times \cdot31831} = 1{\cdot}2407$ ft.;
$1{\cdot}2407^2 \times \cdot7854 = 1{\cdot}53934 \times \cdot7854 = 1{\cdot}209$ sq. ft.

4. Diam. $= 2\sqrt{(1 \div \pi)}$. Cube of diam. $\times \frac{1}{6}\pi = \frac{4}{3}\sqrt{(1 \div \pi)}$
$= \frac{4}{3}\sqrt{\cdot31831} = \cdot5642 \times 1\frac{1}{3} = \cdot7523$ cu. ft.

5. $13^3 \times \cdot5236 = 1150{\cdot}3492$ cu. in. $= \cdot66571$ cu. ft.
$7^3 \times \cdot5236 = 179{\cdot}5948$ cu. in. $= \cdot10393$ cu. ft.
cu. ft. of iron $= \cdot56178$
$(561{\cdot}78 \times 7\frac{1}{2}) \div 16 = 263$ lbs. 5 oz., wt. of iron;
$(103{\cdot}93 \times 1{\cdot}8) \div 16 = 11$ lbs. 11 oz., wt. of powder.

6. $12755^2 \times 12712{\cdot}2 \times \frac{1}{6}\pi$
$= 206814813580 5 \times {\cdot}5236$
$= 1082882364000$ cubic kilomètres ; to which annex nine ciphers for reduction to cubic mètres.

7. $12855^2 \times 12812{\cdot}2 \times \frac{1}{6}\pi$
$= 1108581200000$ cubic kilomètres
1082882364000
25698836000 cubic kilomètres ; to which annex nine ciphers for reduction to cubic mètres.

8. Here $a = 1{\cdot}5$, $b = 3$, $c = 6$; $\therefore \frac{4}{3}\pi abc = 36\pi$
$= 113{\cdot}097$ cu. ft.

9. Here $a = 8$ and $b = 5$; $\therefore \frac{2}{3}\pi ab^2 = 2{\cdot}0944 \times 200$
$= 418{\cdot}888$ cub. in., which $\div 277{\cdot}274$ gives $1{\cdot}51$ gall.

10. If we take a segment of a sphere, radius 1 inch, height of segment $\frac{3}{8}$ of an inch, the cylinder of same height is $\frac{3}{8}\pi$, while the truncated cone is $\frac{1}{3}\pi - \frac{1}{3}\pi(\frac{5}{8})^3 = \frac{120}{512}\pi$; $\therefore$ segment $= (\frac{3}{8} - \frac{120}{512})\pi = \frac{63}{512}\pi$.
Now, the fraction which this is of $\frac{2}{3}\pi = \frac{63}{512} \times \frac{3}{2} = \frac{189}{1024}$; $\therefore$ the volume of the spheroidal segment $= \frac{189}{1024}$ of $1{\cdot}51$ gall. $= {\cdot}28$ gall.

11. Radius being taken as unity, the distances are $\frac{1}{3}$ and $\frac{2}{3}$; $\therefore$ truncated cone $= \frac{1}{3}\pi(\frac{8}{27} - \frac{1}{27}) = \frac{7}{81}\pi$; slice of cylinder $= \frac{1}{3}\pi$; volume of cone $= (\frac{1}{3} - \frac{7}{81})\pi = \frac{20}{81}\pi$; and the fraction which this is of $\frac{4}{3}\pi = \frac{20}{81} \times \frac{3}{4} = \frac{5}{27}$.

12. $\frac{4}{3}\pi(abc - 5^3) = \frac{140}{3}\pi = 146{\cdot}608$ cu. in., which $\div 277{\cdot}274 = {\cdot}53$ gall.

MEASUREMENTS OF CYLINDRICAL SURFACES.

(Page 241.)

1. $3 \times \pi \times 4 = 37{\cdot}7$ sq. in., the curved surface;
$3^2 \times \frac{1}{4}\pi \times 2 = \frac{9}{2}\pi = 14{\cdot}14$ sq. in., area of both ends;
$\therefore$ entire surface $= 51{\cdot}84$ sq. in.

2. $1 \times \pi \times 2 = 6{\cdot}2832$ sq. ft., curved area;
$1^2 \times \frac{1}{4}\pi \times 2 = \frac{1}{2}\pi = 1{\cdot}5708$ cu. ft., contents.

3. The breadth of the belt $= \sqrt{\frac{1}{2}(2)^2} = \sqrt{2}$; but the solidity still $= 1{\cdot}5708$, $\therefore$ area of each end $= 1{\cdot}5708 \div \sqrt{2}$;
$\therefore$ area of both ends $= 1{\cdot}5708 \times \sqrt{2}$
$= 1{\cdot}5708 \times 1{\cdot}4142 = 2{\cdot}2214$
curved area as before $= 6{\cdot}2832$
entire surface $= 8\ 5046$ sq. ft.

4. $27 \times 6 = 162$ sq. in.

5. Taking 6 ft. and 2 ft. as denoting mean diameters, we have—

Outside shell	$6\pi \times 40$	$= 240\pi$ sq. ft.
Flues	$2 \times 2\pi \times 40$	$= 160\pi$
One end, over all	9π	
Deduct for flue ends	2π	
Both ends	$7\pi \times 2$	$= 14\pi$
		414π sq. ft.

This being $\frac{1}{2}$ inch thick makes $414\pi \div 24 = 17\frac{1}{4}\pi$ cu. ft. $= 54{\cdot}1926$ cu. ft., which increased by 10 per cent. gives $59{\cdot}6119$ cu. ft. of iron. A cubic foot of iron weighs 480 lbs.; hence $59{\cdot}6119 \times 480 = 28614$ lbs. $= 12$ t. 15 cwt. 1 qr. 26 lbs.

6. Cubic ft. of metal $= \{4^2 - (3\frac{3}{4})^2\} \times \frac{1}{4}\pi \times 12 \div 1728$
$= 31 \times 3\pi \div (1728 \times 16)$;
wt. in lbs. $= \frac{31 \times 3\pi \times 8{\cdot}8 \times 62\frac{1}{2}}{1728 \times 16} = \frac{31 \times 3\pi \times 550}{1728 \times 16}$,
$= 5{\cdot}805$ lbs. at $7\frac{1}{2}d. = 3s.\ 7\frac{1}{2}d.$

MEASUREMENTS OF CONICAL SURFACES.

(Page 243.)

1. Area of base $= 10^2 \times \pi$; curved area $= 20 \times 10 \times \pi$;
$\therefore$ whole surface $= 300\pi = 942{\cdot}48$ sq. ft.

2. Slant height $= \sqrt{20^2+15^2} = 25$ in. $\therefore$ curved surface $= \pi \times 25 \times 20 = 1570{\cdot}8$ sq. in.

3. Curved surface $= \frac{1}{2}(7\pi+10\pi) \times 5 = \frac{1}{2}\pi \times 85 = 133{\cdot}5$ sq. in.

4. Slant height $= \sqrt{5^2+12^2} = 13$;
curved surface $= \frac{1}{2}(8\pi+18\pi) \times 13 = 169\pi$;
areas of ends $= (4^2+9^2) \times \pi = 97\pi$;
$\therefore$ whole area $= 266\pi = 835{\cdot}67$ sq. in.

5. Slant height $= \sqrt{20^2+10^2} = 22{\cdot}36$ in.; sum of top and bottom radii $= 30$ in.;

curved surface	$= 30\pi \times 22{\cdot}36$	$= 670{\cdot}8\pi$
bottom area		$= 100{\cdot}0\pi$
flange	$= (44^2-40^2) \times \frac{1}{4}\pi$	$= 84{\cdot}0\pi$
		$854{\cdot}8\pi$ sq. in.

$854{\cdot}8\pi \times \frac{5}{16} = 266\frac{7}{8}\pi$ cu. in. of iron $= 838{\cdot}414$ cu. in. $= {\cdot}485$ cu. ft. of iron at $62\frac{1}{2} \times 7{\cdot}7$ lbs. per foot $= 233{\cdot}4$ lbs.

6. Slant height $= \sqrt{(7\frac{1}{2})^2+20^2} = 21{\cdot}36$;

curved surface	$= \frac{1}{2}(15 \times \pi \times 21{\cdot}36)$	$= 160{\cdot}2\pi$
area of base	$= 15^2 \times \frac{1}{4}\pi$	$= 56{\cdot}25\pi$
		$216{\cdot}45\pi$

Let r denote radius of required circle;
$r^2 \times \pi = 216{\cdot}45\pi$; $\therefore r = \sqrt{216{\cdot}45} = 14{\cdot}71$ in.

7. *See* Answer in Text-Book.

MEASUREMENTS OF SPHERICAL SURFACES.

(Page 246.)

1. $12^2 \times \pi = 452{\cdot}39$ sq. in.

2. $\frac{1}{2}\{(48\frac{3}{16})^3 - (47\frac{13}{16})^3\} \times {\cdot}5236 \times \frac{8900}{16 \times 1728}$

$= \frac{(111893 - 109301) \times 52{\cdot}36 \times 89}{2 \times 16 \times 1728} = \frac{3 \times 13{\cdot}09 \times 89}{16}$

$= 218{\cdot}44$ lbs. $= 218$ lbs. 7 oz.

3. $5 \times \pi \times \frac{1}{4} = {\cdot}7854 \times 5 = 3{\cdot}927$ sq. ft.

4. The curved surface of a zone of a sphere = the product of the height of the zone by the circumference of the sphere. Let h denote the height, and d the diameter of the sphere; surface $= d \times \pi \times h$; a mean proportional between d and $h = \sqrt{(d \times h)}$; and area of circle whose radius is $\sqrt{(d \times h)}$ is $d \times h \times \pi$.

5. Radius $= \sqrt{5 \times \frac{1}{4}} = \frac{1}{2}\sqrt{5} = 1{\cdot}118$ ft.

6. No. of square inches in the earth's surface

$= 41777600^2 \times 3{\cdot}1416 \times 144$;

no. of tons required $= \frac{3}{100}$ of $\frac{41777600^2 \times 3{\cdot}1416 \times 144 \times 14{\cdot}7}{2240}$

$= \frac{1}{3}(3 \times 417776^2 \times 78{\cdot}54 \times 18{\cdot}9) = 155450072000000$ tons.

7. Difference of cubes of outer and inner diameters

$= \frac{20 \times 1728}{8{\cdot}9 \times 62{\cdot}5 \times {\cdot}5236} = \frac{8000 \times 1728}{89 \times 1309} = 118{\cdot}66$ cub. in.

$\therefore$ cube of inner diam. $= 12^3 - 118{\cdot}66 = 1609{\cdot}34$;

$\sqrt[3]{1609{\cdot}34} = 11{\cdot}7188$ in.; $\therefore$ the thickness

$= \frac{1}{2}(12 - 11{\cdot}7188) = {\cdot}1406$ of an inch.

8. The segment is $\frac{11}{256}$ of the sphere; the whole spherical shell would therefore weigh $\frac{5120}{11}$ lb., and would measure $\frac{5120}{11 \times 8{\cdot}9 \times 62\frac{1}{2}} = \frac{512 \times 8}{55 \times 89}$ cu. ft. This, at $\frac{1}{8}$ of an inch, or $\frac{1}{96}$ of a foot thickness will have a surface very nearly $= \frac{512 \times 8 \times 96}{55 \times 89}$ sq. ft.; whence radius of sphere

$$= \sqrt{\frac{512 \times 8 \times 96}{55 \times 89 \times 4\pi}} = \sqrt{\frac{512 \times 80000}{55 \times 89 \times 1309}} = \sqrt{6{\cdot}392}$$
$$= 2{\cdot}528 \text{ ft.}$$

Radius of segment = radius of sphere $\times \sqrt{(1 - \frac{9}{16})}$ $= {\cdot}632 \times 2{\cdot}6457 = 1{\cdot}672$; $\therefore$ diam. of segm. $= 3{\cdot}344$ ft.

Papers set at Public Examinations.

ROYAL SCHOOL OF NAVAL ARCHITECTURE.

(Page 249.)

1. 5 lbs. $\times 60 \times 36$, at $3\frac{1}{2}d. = 300 \times 126d. = £157\ 10s.$ *Ans.*

2. $7{\cdot}492 : 45 \div 62\frac{1}{2} :: 4$ in. thick : $\frac{2880}{7492} = {\cdot}3844$ in. *Ans.*

3.

1500 tons iron	@ £16¼ =	£24375
75 „ „	@ £26½ =	1987·5
640 „ „ castings	@ £14 =	8960
45 „ gun metal	@ 11*d.* a lb. =	4620
27*s.* $\times 120 \times 52$ = cost of labour	=	8424
1200 tons coal	@ 13*s.* =	780
Superintendence, interest, &c.	=	4000
		£53146·5
Contingencies, $\frac{1}{10}$	=	5314·65
		58461·15
Proposed profit 40%	=	23384·46
Tons 2260)		81845·61
		£36·215

$= £36\ 4s.\ 4d.$ *Ans.*

4. G.C.M. = 23; whence $\frac{161}{247}\frac{23}{370}\frac{37}{} = \cdot 66037$. *Ans.*

5. The slant height of each end = $\sqrt{20^2+15^2} = 25$ ft., which is also the slant height of each side, as the difference of the length over all and the ridge = the breadth over all. Thus the whole surface of the roof = $(30+90)\times 25$ = 3000 sq. ft. *Ans.*

Again; the cubical content is that of a wedge, the length of the base of which is 60, that of the edge 30, the breadth of the base 30, and the height 20; ∴ the cubical content is $\frac{1}{3}(30+120)\times\frac{1}{2}(30\times 20) = 50\times 300 = 15000$ cu. ft. *Ans.*

6. $(35^2-33^2)\times\frac{1}{4}\pi = 68\times 2\times\frac{1}{4}\pi = 34\pi$ sq. in.
$= (17\pi\div 72)$ sq. ft.; which $\times 5280\times 4$ gives
$\frac{1}{3}(17\pi\times 880)$ cu. ft. of piping; which

$$\times\frac{105\times 62\frac{1}{2}\times 7\frac{1}{4}\times 9}{100\times 2240} \text{ gives } \frac{17\pi\times 110\times 105000\times 261}{3\times 100\times 2240\times 8}$$

$$= \frac{17\pi\times 55\times 5\times 261}{128} = £29947{\cdot}594$$

$= £29947$ 11*s.* 10½*d.* *Ans.*

7. The sum of the areas of the two triangles into which either of the diagonals divides the quadrilateral = 80×116 = 9280 sq. ft. = $1031\frac{1}{9}$ sq. yds. *Ans.*

8. Volume of the glass : volume of water :: 2 : 1, or 8 : 4; hence $\sqrt[3]{8} : \sqrt[3]{4} :: 2\frac{1}{2}$ in. : $1\frac{1}{4}\times\sqrt[3]{4} = 1\frac{1}{4}\times 1{\cdot}5874$ = 1·98425 inch, depth of water. *Ans.*
Quantity = ½ of $(2^2\times{\cdot}7854\times\frac{1}{3}$ of $2\frac{1}{2}) = 1{\cdot}309$ cu. in. *Ans.*

9. $\frac{5}{13}$ of $\frac{39}{7} = \frac{15}{7}$; and $\frac{3}{5}$ of $\frac{5}{4}$ of $\frac{7}{3} = \frac{7}{4}$;

$$\frac{15}{7}-\frac{7}{4} = \frac{60-49}{28} = \frac{11}{28}. \quad \textit{Ans.}$$

Also $\frac{2\frac{1}{4}}{9\frac{3}{7}} = \frac{9\times 7}{66\times 4} = \frac{21}{88}$; and $\frac{21}{88}\times\frac{11}{3} = \frac{7}{8}$. *Ans.*

ENGINEER STUDENTS OF THE ADMIRALTY.

(Page 250.)

4. *Ans.* 1512632327.

5. *Ans.* £108920 11*s.* 6*d.*

6. £824019 8*s.* $4\frac{1}{2}$*d.*
38274 13 $9\frac{3}{4}$
Ans. £785744 14 $6\frac{3}{4}$

7. *Ans.* 570713051852.

8. *Ans.* £648467 16*s.* 11*d.*

9. *Ans.* £12447 5*s.* $8\frac{3}{4}$*d.*

10. *Ans.* £375623 17*s.* $11\frac{3}{4}$*d.*

11. *Ans.* 85296 oz.

12. 2 : 13 :: £6 12*s.* $8\frac{1}{2}$*d.*
$6\frac{1}{2}$
Ans. £43 2 $7\frac{1}{4}$

13. £2870 10*s.* @ 14*s.* 3*d.* in the pound

10*s.*	$\frac{1}{2}$	1435	5	
4*s.*	$\frac{1}{5}$	574	2	
3*d.*	$\frac{1}{40}$	35	17	$7\frac{1}{2}$
		£2045	4	$7\frac{1}{2}$

Ans.

14. $\frac{2\frac{1}{2}\times 4}{100}$ or $\frac{1}{10}$ of £3260 = £326. *Ans.*

15. 14175*d.* ÷ 5*d.* = £2835. *Ans.*

16. *Ans.* 59 ac. 14 po. 13 yds. 5 ft. 124 in.

17. *Ans.* $7\frac{101}{120}$.

18. *Ans.* $1\frac{69}{80}$.

19. *Ans.* $1\frac{87}{112}$.

20. *Ans.* $25\frac{38}{88}$.

21. *Ans.* 1150·74*d.*

22. *Ans.* 11 oz. 19 dwt. 23·28 grs.

23. *Ans.* ·0886212.

24. *Ans.* ·003125.

25. *Ans.* 130.

26. $13\frac{1}{8} \div 60 = 13{\cdot}125 \div 60 = {\cdot}21875$. *Ans.*

27.
$$\left.\begin{array}{rcr} 90 & : & 200 \\ 10 & : & 8 \\ 160\times6\times4 & : & 450\times4\times3 \end{array}\right\} :: 12 \text{ da.} : 30 \text{ da. } \textit{Ans.}$$

28. 300 francs : 25 francs :: 135 guld. : $11\frac{1}{4}$ guld.
$20\frac{1}{2}d. \times 11\frac{1}{4} = 230\frac{5}{8}d.$, or $9\frac{3}{8}d.$ loss on every sovereign. *Ans.*

240 : $9\frac{3}{8}$ :: 25 francs : 98 centimes, or nearly 1 franc on every 25 francs. *Ans.*

29. $\sqrt{531441}$ sq. yds. = 729 yds. *Ans.*

30.
$$\begin{array}{rcl} 4 \text{ mi.} \div 3\frac{3}{4} \text{ mi.} & = & 1\frac{1}{15} \text{ hr.} \\ 2 \text{ mi.} \div 3\frac{1}{3} \text{ mi.} & = & \frac{3}{5} \text{ hr.} \\ 1 \text{ mi.} \div 3 \text{ mi.} & = & \frac{1}{3} \text{ hr.} \\ \hline 7 \text{ miles walked in } 2 & & \text{hours.} \end{array}$$

∴ he must walk 3 miles an hour for the remainder of the journey. *Ans.*

ADDITION PAPER.

(Page 252.)

	£	s.	d.		£	s.	d.
(1)	61332	17	1	(2)	65189	0	6
(3)	47301	7	5	(4)	47647	12	6
(5)	39684	15	2	(6)	65650	7	2
(7)	50597	1	3	(8)	34626	1	0
(9)	34081	1	4	(10)	43989	13	6
(11)	28339	16	11	(12)	53462	12	9

OXFORD LOCAL EXAMINATIONS.

(Page 254.)

1. *Ans.* 698778. 2. *Ans.* 91132497; and 506909.

3. *Ans.* £1111 11*s.* 11¾*d.*; and 1 t. 16 cwt. 3 qrs. 23 lbs.

4. *Ans.* £158 6*s.* 10¼*d.*

6. *Ans.* 418 t. 2 qrs. 12 lbs. 15 oz.

7. *Ans.* £2 2*s.* 2¼*d.*; also ⅓ of £99, &c. = £33 0*s.* 11¼*d.*

5. 1 lb. av. = 7000 grs.
1 lb. tr. = 5760 grs.
∴ 1 lb. av. = $\frac{7000}{5760}$ lb. tr.
or 5760 lb. av. = 7000 lb. tr.
or 144 lb. av. = 175 lb. tr.

8. 5280 ft. × 3 ÷ 7⅓ ft. = 2160 times. *Ans.*

9. 45 ft. : 61 ft. :: 405*d.* : 9*d.* × 61 = 549*d.*
= 45*s.* 9*d.* *Ans.*

10. 4000 bu. at 7*s.* a bu. = 28000*s.*
Proposed profit = 500
28500
2000 bu. at 6*s.* a bu. = 12000
16500*s.* ÷ 2000 = 8*s.* 3*d.* *Ans.*

11. 45 in. × 3·1416 = 141·37 in. = 11 ft. 9·37 in. *Ans.*

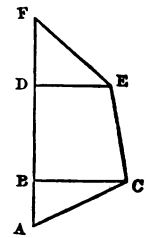
12. The perpendicular altitude of the equilateral triangle forming the base of the pyramid = 3√3; ∴ its area = 9√3; and the volume of the pyramid is 9√3 × ⅓ of 60 = 180√3 = 17·3205 × 18 = 311·769 cu. ft. *Ans.*

13. The area of the ring = $(525^2 - 482^2) \times 7853982$ sq. ft.
= 1007 × 43 × ·7853982 = 34008·52746 sq. ft.
= 3778·72527 sq. yds. @ 2*s.* 6*d.* = £472·34066,
or £472 6*s.* 9¾*d.* *Ans.*

14. Surface of globe $= 18 \times 18 \times 3{\cdot}14159 = 1017\frac{7}{8}$ sq. in. *Ans.*

15. $4 \times 2\frac{1}{2} \times 3\frac{1}{4} \times 1000$ oz. $= (13 \times 2500) \div 16$ lbs.
$= 2031\frac{1}{4}$ lbs. $= 18$ cwt. $15\frac{1}{4}$ lbs. *Ans.*

17. AB 575, BD 1290, DF 853,
BC 1142, DE 953.



Areas :—

ABC	$= 575 \times 571$	$= 328325$
BE	$= 645(953+1142)$	$= 1351275$
DE	$= \frac{1}{2}(853 \times 953)$	$= 406454$
	whole area	2086054. *Ans.*

CAMBRIDGE LOCAL EXAMINATIONS.—(Junior).

(Page 255.)

1. *Ans.* 22265; which $\div 365$ gives 61.

2. *Ans.* £36 8*s.* $2\frac{1}{2}$*d.*

3. 128275 lbs. $\div$ 3665 lbs. $= 35$ lots. *Ans.*

4. *Ans.* ·718; also ·175 of £1 $= 3s.\ 6d.$; and 1·0037 t. $=$ 1 ton 8 lbs. $6\frac{2}{5}$ oz.

5. *Ans.* $1\frac{1}{12}$; also $\frac{88}{21} + \frac{39}{4} = \frac{1171}{84} = 13\frac{79}{84}$. *Ans.*

6. $4(33-25)+25 = 57$. *Ans.*

7. 5 cwt. 3 qrs. 19 lbs. @ £ 2 15*s.* per cwt.

		2	15		×3
		8	5		
3 qrs.	$\frac{1}{4}$	2	1	3	
12 lbs.	$\frac{1}{7}$	0	5	$10\frac{5}{7}$	
7 lbs.	$\frac{1}{12}$	0	3	$5\frac{1}{4}$	
		£16	5	7	*Ans.*

8. $1609\frac{1}{2}s. : 20s. \times 407 \\ 12 \text{ da.} : 54 \text{ da.}$ $\Big\}$ :: 29 m. : 660 m. *Ans.*

9. Paid altogether $359d.$
$374 @ \frac{1}{2}d. = 187$
$172d. + \frac{2}{3}d. = 258$ eggs. *Ans.*

10. The whole sum equals $1+2+3 = 6$ times A's share;
∴ A gets £5 17*s.* $6\frac{1}{4}d.$, B £11 15*s.* $0\frac{1}{2}d.$,
C £17 12*s.* $6\frac{3}{4}d.$ *Ans.*

11. $(5d. + 27d. + 48d. + 48d.) \div (1+3+4+12) = 6\frac{3}{8}d.$ *Ans.*

12. $\frac{4700}{98} \times 3\frac{1}{2} =$ £167 17*s.* $1\frac{5}{7}d.$
$\frac{4700}{94} \times 3 =$ 150 0 0
Ans. £17 17 $1\frac{5}{7}d.$ to his advantage.

13. *Ledger.*

Dr.	Stock Account		Cr.
To balance	£225	By Cash	£200
		„ Profit & Loss	25
	225		225

Dr.	A. B. Etc.		Cr.
To Balance	280	By Goods	280

Dr.	C. D. Etc.		Cr.
To Goods	330	By Cash	120
		„ Balance	210
	330		330

Dr.	Cash Account		Cr.
To Stock	200	By Profit & Loss	25
„ C. D. &c.	120	„ Balance	295
	320		320

Dr.	Goods Account		Cr.
To A. B. &c.	280	By C. D. &c.	330
„ Profit & Loss	50		
	330		330

Dr.	Profit & Loss Account		Cr.
To Cash	25	By Goods	50
„ Stock	25		
	50		50

Dr.	Balance Account		Cr.
To C. D. &c.	210	By A. B. &c.	280
„ Cash	295	„ Stock	225
	505		505

CAMBRIDGE LOCAL EXAMINATIONS.—(Senior).

(Page 256).

1. *Ans.* 1 fur. 5 po. 2 yds. 2 ft. $8\frac{23}{52}$ in.

2. (i) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7} = \frac{5}{4}+\frac{3}{2}+\frac{4}{5}+\frac{6}{7} = \frac{617}{140}$,
and $1\frac{10001}{100}-\frac{617}{140} = \frac{33461}{350} = 95\frac{211}{350}$. *Ans.*

(ii) $\frac{1}{2+\frac{4}{13}} = \frac{13}{30}$. *Ans.*

(iii) $\frac{1}{3}$ of $\frac{1}{3}\div(\frac{7}{6}+\frac{20}{3}) = \frac{1}{6}\div\frac{47}{6} = \frac{1}{47}$. *Ans.*

(iv) $\frac{28\frac{4}{7}-21}{47\frac{4}{7}+10} = \frac{7\frac{4}{7}}{57\frac{4}{7}} = \frac{39\times7}{403\times5} = \frac{3\times7}{31\times5} = \frac{21}{155}$. *Ans.*

3. (i) $100000-54183{\cdot}88521 = 45816{\cdot}11479$. *Ans.*

(ii) $\cdot2\dot{1}\div\cdot2\dot{1}\dot{1} = \frac{21}{99}\times\frac{999}{211} = \frac{2331}{2321}$.

4. $2\div\cdot2 = 20\div2 = 10$. *Ans.*; also $\cdot002\div\cdot02 = \cdot2\div2 = \cdot1$. *Ans.*; and $2{\cdot}2+2{\cdot}1 = 22+21 = 1\frac{1}{21}$. *Ans.*

5. $\frac{1}{4}$ of 17*s.* 8*d.* + $2\frac{5}{8}$ of 1*s.* = 4*s.* 5*d.* + 2*s.* $7\frac{1}{2}$*d.* =

	s.	*d.*
	7	$0\frac{1}{2}$
	6	7
	13	$7\frac{1}{2}$
$\frac{15}{32}$ of 5*s.* 4*d.* = 30*d.* =	2	6
Ans. Value	11*s.*	$1\frac{1}{2}$*d.*

$11\frac{1}{8}s. \div 100s. = 89 \div 800 = .11125$. *Ans.*

6. $\sqrt{35.672} = 5.9726$. *Ans.*
$\sqrt{\frac{4}{9}} = \frac{2}{3} = .6667$. *Ans.*

7. $\frac{150 \times 50 \times 5}{\frac{3}{4} \times 12} = \frac{50 \times 250}{3} = £4166\ 13s.\ 4d.$ *Ans.*

8. (i) $£5 \times 5 = £25$ the int.
125 : 25 :: 100 : 20 the disc.
$£5$ *Ans.*

(ii) Int. on £200 for 2 years = £18; ∴ int. on £100 for 1 year = $£4\frac{1}{2}$;
127 : 100 :: £1000 : £787 8*s.* $0\frac{48}{127}$*d.* *Ans.*

9. Since $\frac{1}{3}$ are over 15, and $\frac{1}{3}$ are between 10 and 15, the rest are $\frac{1}{3}$; and hence 2000*s.* ÷ (10*s.* + 6*s.* 8*d.* + 3*s.* 4*d.*) = 100 boys of each class; ∴ 300 boys in all. *Ans.*

10. $8\frac{1}{2}$ in. : $10\frac{1}{4}$ in. :: 9*d.* : $10\frac{29}{34}$*d.*; hence the $8\frac{1}{2}$ inch candles are cheaper by $\frac{5}{34}$ of a penny per half-dozen.
Also 11*d.* : $\frac{5}{34}$*d.* :: 100 : 1.3369 per cent. saved. *Ans.*

Training Schools.

CANDIDATES FOR ADMISSION.

(Page 257.)

Section I.

1. 2238883 − 598799 = 1640084. *Ans.*

2. *Ans.* 124, and 26872 rem^{r}.

Section II.

1. *Ans.* 57291 farthings.

2. 10000000 ÷ (8766 × 60) = 19 yrs. 6760 min.
= 19 yrs. 4 da. 16 hrs. 40 min. *Ans.*

Section III.

1. 2 sec. : 24 × 60 × 60 sec. :: 5 ti. : 216000 ti. *Ans.*

2. 52 wks. : 7 wks. :: 208*s.* : 28*s.* *Ans.*

Section IV.

1.

		£	2	12*s.*	6*d.*
					23
		60	7	6	
2 ro.	$\frac{1}{2}$	1	6	3	
1 ro.	$\frac{1}{2}$	0	13	$1\frac{1}{2}$	
16 po.	$\frac{1}{10}$	0	5	3	
		£62	12	$1\frac{1}{2}$	*Ans.*

2. $\frac{12\frac{1}{3} \times 12\frac{1}{2} \times 4\frac{1}{2}}{9 \times \frac{3}{4}}$

$= \frac{37s. \times 25}{9}$

$= £5\ 2s.\ 9\frac{1}{3}d.$ *Ans.*

Section V.

1.

$3\frac{1}{2}$ cwt. coals	@ $10\frac{1}{2}d.$	£0	3	$0\frac{3}{4}$
13 lbs. cheese	@ $7\frac{3}{4}d.$	0	8	$4\frac{1}{4}$
$2\frac{3}{4}$ lbs. tea	@ 3*s.* 4*d.*	0	9	2
17 lbs. sugar	@ $5\frac{1}{2}d.$	0	7	$9\frac{1}{2}$
$3\frac{1}{2}$ yds. flannel	@ 1*s.* $11\frac{1}{2}d.$	0	6	$10\frac{1}{4}$
29 yds. calico	@ $10\frac{3}{4}d.$	1	5	$11\frac{3}{4}$
		£3	1	3

£5 − £3 1*s.* 3*d.* = £1 18*s.* 9*d.*, change out.

2. The fittings cost 1 sum, and the yacht 5 times that sum; so the whole = 6 times the cost of the fittings; hence, £1005 12*s.*÷6 = £167 12*s.* for the fittings; ∴ the yacht cost £838. *Ans.*

SECTION VI.

1. Cost
25 p. c. = $\frac{1}{4}$ = £30 12*s.* 6*d.*
7 13 1½
£38 5 7½÷1225 lbs. = 7½*d.* per lb. *Ans.*

2. 32725*s.*÷(42*s.*+21*s.*+14*s.*) = 425 qu. of each grain. *Ans.*

SECTION VII.

1. $\frac{(15+36+25+21)\times 40}{(20+24+35+36)\times 45} = \frac{97\times 8}{115\times 9} = \frac{776}{1035}$ *Ans.*

2. $\frac{5}{6}-\frac{2}{3} = \frac{1}{6}$ of the capacity = 50 gallons; ∴ the cistern holds 300 gallons. *Ans.*

SECTION VIII.

1.
5·384×·00723 = ·03892632
20·7×500 = 10350·0
50·34×·072 = 3·62448
10353·66340632
$\frac{1}{8}$ of ·462 = ·0924
Ans. 10353·57100632

2. 500÷·25 = 50000÷25 = 2000
2000÷·025 = 2000000÷25 = 80000
80000÷50 = 1600 *Ans.*

Training Schools.

CANDIDATES OF FIRST YEAR.

(Page 259.)

SECTION I.

1. $7051968 - 397897 = 6654071$. *Ans.*

2. $365 \times 10 + 3$ leap year da. $= 3653$ da. $= 87672$ hrs. *Ans.*

3. £3698 16*s.* : £613 2*s.* 8*d.* :: 20*s.* : 3*s.* $3\frac{72}{92}\frac{47}{47}d.$ *Ans.*

4.

7 lbs. @ 2*s.* 3*d.*	£0	15	9*d.*
16 lbs. @ 7*d.*	0	9	4
11 oz. @ 1*s.* 7*d.*	0	17	5
9 lbs. @ $3\frac{1}{3}d.$	0	2	$7\frac{1}{2}$
7 lbs. @ $7\frac{3}{4}d.$	0	4	$6\frac{1}{4}$
$3\frac{1}{2}$ lbs. @ 1*s.* 10*d.*	0	6	5
	£2	16*s.*	$0\frac{3}{4}d.$

5. 2016 dwt. ÷ 56 dwt. = 36 spoons. *Ans.*

6. 10 galls. @ 12*s.* + 15 galls. @ 4*s.* 6*d.* + 18 galls. @ 5*s.* 9*d.* + 45*s.* 6*d.* = 336*s.* 6*d.*, which ÷ 43 gives 7*s.* $9\frac{32}{43}d.$ *Ans.*

SECTION II.

1. 9 : 148995 :: 5 : 82775. *Ans.*

2. $\left.\begin{matrix} 4 : 5 \\ 14\frac{1}{4} : 19 \end{matrix}\right\}$ :: 6 wks. : 10 wks. *Ans.*

3. $\frac{1}{2} \times \frac{6+4+3}{63+45+35} \times \frac{315}{12} = \frac{13}{143} \times \frac{315}{24} = \frac{1}{11} \times \frac{105}{8} = 1\frac{17}{88}$. *Ans.*

SECTION III.

1. $\frac{382}{125} \times \frac{25}{7} \times \frac{345}{47 \times 9} = \frac{8786}{987}$. *Ans.*

2. $\frac{5}{7} : \frac{2}{5} :: £1003\ 17s.\ 1d. : £562\ 3s.\ 2d.$ *Ans.*

3. $\frac{3}{8}$ of $£\frac{1}{8} + \frac{4}{5}$ of $£\frac{21}{20} + £\frac{5}{3} - \frac{1}{10}$ of $£\frac{1}{240}$
$= £\frac{3}{64} + £\frac{84}{100} + £\frac{1333}{800}$,
$= £\cdot 046875 + £\cdot 84 + £1\cdot 66625 = 2\cdot 553125$ of £1
$= \cdot 510625$ of £5. *Ans.*

SECTION IV.

1. $(\cdot 625 + \cdot 4375 + \cdot 75 + \cdot 09375 + 2\cdot 46) \times \frac{1}{125}$
$= 4\cdot 36625 \times \cdot 008 = \cdot 03493$. *Ans.*

2. 14·5 galls. ÷ 36 galls. = ·402$\dot{7}$. *Ans.*

also, $\frac{62\cdot 25 \times 25}{28 \times 24 \times 3600} = 83 \div 129024 = \cdot 000643$. *Ans.*

3.

		£111 11*s.* 4*d.*
		45
		5020 10 0
2 ro.	$\frac{1}{2}$	55 15 8
1 ro.	$\frac{1}{9}$	27 17 10
20 per.	$\frac{1}{2}$	13 18 11
		£5118 2*s.* 5*d.* *Ans.*

SECTION V.

1. $\frac{500}{1350}$, or $\frac{10}{27}$, of £375 = £138 17*s.* 9$\frac{1}{3}$*d.*
$\frac{850}{1350}$, or $\frac{17}{27}$, of £375 = £236 2*s.* 2$\frac{2}{3}$*d.* } *Ans.*

2. 5 guin. : 4$\frac{1}{2}$ guin. :: 105 : 94$\frac{1}{2}$;
100 − 94$\frac{1}{2}$ = 5$\frac{1}{2}$ per cent. loss. *Ans.*

3. 5 h. = 84 sh.; ∴ 15 h. = 252 sh., and 10 h. = 168sh.; hence, $\left.\begin{matrix}400 \text{ sh.} : 300 \text{ sh.} \\ 9 \text{ da.} : 8 \text{ da.}\end{matrix}\right\} :: £75\frac{3}{4} : £50\ 10s.$ *Ans.*

Section VI.

1. $4 \times 2\frac{1}{2} \times 1\frac{1}{4} \times 1728 : 100 :: 27 \text{ cwt.} : \frac{27 \times 100 \times 4}{10 \times 5 \times 1728}$ $= \frac{1}{8}$ cwt. = 14 lbs. *Ans.*

2. $\sqrt{4020025} = 2005$. *Ans.*; also $\sqrt{\cdot 000009} = \cdot 003$. *Ans.* and $\sqrt{\cdot 90} = \cdot 94868$. *Ans.*

3. His power of rowing in still water is 3 mi. an hour. But, with a current of $1\frac{1}{2}$ mi. an hour in his favour, he would row from A to B in $1\frac{1}{2} \div (3 + 1\frac{1}{2}) = \frac{1}{3}$ of an hour = 20 min.; and with the same current against him, he would row from B towards A at the rate of $3 - 1\frac{1}{2}$ or $1\frac{1}{2}$ mi. an hour, which in 10 min. would make $\frac{1}{6}$ of $1\frac{1}{2}$ mi. or $\frac{1}{4}$ of a mile; then the remaining $1\frac{1}{4}$ mi., at the rate of 3 mi. an hour, would take $\frac{5}{12}$ of an hour or 25 min.; so that the time required is 30 + 25 = 55 min. *Ans.*

Section VII.

1. $4\frac{3}{8} : 4 :: 100 : 91\frac{3}{7}$. *Ans.*

2. Income tax = $7d. \times 200 = 1400d.$ to be saved in sugar, by the reduction of $1\frac{1}{2}d.$ a lb.; ∴ $1400 \div 1\frac{1}{2} = 933\frac{1}{3}$ lbs. = $8\frac{1}{3}$ cwt. *Ans.*

3. B has 250 yds. to gain at the rate of $\frac{1}{3}$ of a yard per minute, which will take 750 min.; and in that time A will have gone $11 \times 750 = 8250$ yds.; that is, he will have gone $8250 \div 476 = 17\frac{79}{238}$ times round. *Ans.*

Training Schools.

CANDIDATES OF SECOND YEAR.

(Page 261.)

1. 110 : 10 :: 50*d.* : $4\frac{6}{11}$*d.* gain per lb.
   £12 ÷ $4\frac{6}{11}$*d.* = 3168 ÷ 5 = $633\frac{3}{5}$ lbs. = 5 cwt. $73\frac{3}{5}$ lbs. *Ans.*

2. He bought in at $79\frac{5}{8}$, and sold out for £2061,
   ∴ 1911 : $79\frac{5}{8}$ :: 2061 : $85\frac{7}{8}$;
   ∴ $85\frac{7}{8} + \frac{1}{8} = 86$. *Ans.*

3. $\frac{\cdot 79583}{8\cdot 79583} + \frac{5\cdot 20417}{14\cdot 79583} = \cdot 0905 + \cdot 3517 = \cdot 44$. *Ans.*

4.

4	1·00	
2	·25	× 1 = ·25
2	·125	× 2 = ·25
2	·0625	× 3 = ·1875
2	·03125	× 4 = ·1250
2	·015625	× 5 = ·078125
	·0078125	× 6 = ·046875
		·9375

5 What decimal fraction of $\frac{5}{7}$ of 11·718$\frac{3}{4}$ measures is equal to $\frac{3}{5}$ of 9·166$\frac{2}{3}$ such measures?

$$\frac{\frac{3}{5} \text{ of } 9166\frac{2}{3}}{\frac{5}{7} \text{ of } 11718\frac{3}{4}} = \frac{5500 \times 7 \times 4}{46875 \times 5} = \frac{220 \times 28}{9375}$$

= ·65706. *Ans.*

6. The estate yields £1000 per ann.
   4 per cent. on £15000 = £600
   Cost of repairs 180
   Net produce, £220 per ann.
   780
   5000 : 220 :: 100 : $4\frac{2}{5}$ per cent. per ann. *Ans.*

7. 15 lbs. × 144 × 14 = 30240 lbs.
$\frac{1}{30}$ = 1008
31248 lbs. = 13 t. 19 cwt. *Ans.*

8. 90 : 128 :: $7\frac{1}{2}$ per cent. : $10\frac{2}{3}$ per cent. *Ans.*

9. The value of the pearls is a mean proportional between the values of the casket and the diamonds;

$\therefore \sqrt{68229 \times 21} = 1197s. = £59\ 17s.$ *Ans.*

10. $25{\cdot}20 \times \frac{8\frac{1}{3}}{20} = 10\frac{1}{2}$ guld. $10\frac{1}{2} \div 11\frac{2}{3} = £\frac{9}{10}$, showing a loss of 10 per cent. *Ans.*

11. The area, = $1496\frac{1}{3}$ sq. po., consists of 3 squares each $498\frac{7}{9}$ sq. po.; ∴ the square root of this = $\frac{1}{3}\sqrt{4489}$ = $22\frac{1}{3}$ po. = $122\frac{5}{6}$ yds. the breadth of the piece of ground, and $368\frac{1}{2}$ yds. the length.

Hence the circuit = $(1+3) \times 2 \times 122\frac{5}{6} = 982\frac{2}{3}$ yds. *Ans.*

Also, the diagonal = $\sqrt{(3^2+1^2)} \times 122\frac{5}{6}$
= $3{\cdot}162277 \times (123 - \frac{1}{6}) = 388{\cdot}433$ yds. *Ans.*

Again: the mètre being equal to 3·2809 Eng. ft., the corresponding metric question is:—A rectangular piece of ground of 3 hectares 78 ares 45 sq. mètres is one-third, &c., &c.

The length will be found 336·95 mèt., and the breadth 112·32 mèt.; the circuit 898·53 mèt.; and the diagonal = 3·16227 × 112·32 = 355·18 mèt.

12.

5	1·0		
10	·2	× 2 =	·4
15	·02	× 4 =	·08
20	·00133333	× 8 =	·0106667
25	·00006666	× 16 =	·0010667
30	·00000266	× 32 =	·0000853
	·00000008	× 64 =	·0000056
			·491824 *Ans.*

13. The sum of their velocities = (92+84) ft. in $1\frac{1}{2}$ sec., = 80 miles an hour; the difference of their velocities = (92+84) ft. in 6 seconds = 20 miles an hour;

hence, $\frac{1}{2}(80+20)$ = 50 mi. an hour,
and $\frac{1}{2}(80-20)$ = 30 mi. an hour. } *Ans.*

Society of Arts.

ARITHMETIC.

(Page 262.)

1. $\frac{600 \text{ cwt.} - 2\frac{1}{4} \text{ cwt.}}{7 \text{ cwt. } 13 \text{ lbs.}} = \frac{66948}{797} = 84$; hence 85 casks. *Ans.*

2. Selling price = $\frac{18}{100}d. \times 144$ per gross;
profit = $(\frac{18}{100}d. \times 144 - 22d.) \times 25$,
$9d. \times 72 - 550d. = 98d. = 8s.\ 2d.$ *Ans.*

3. £14 6*s.* + £8 16*s.* = twice the worth of the watch;
∴ £23 2*s.* ÷ 2 = 11 guineas for the watch,
£2 15*s.* for the chain. } *Ans.*

4. 3 cwt. @ 36*s.* 6*d.* = 109*s.* 6*d.*; which ÷ $4\frac{1}{2}d.$ gives 292 lbs. sold; ∴ 336 − 292 = 44 lbs. reserved. *Ans*

5. $3\frac{3}{4}$ mi. × $\frac{1}{3}$ of an hour = $1\frac{1}{4}$ mi., the distance of A in advance of B, which B is to gain at the rate of $8\frac{1}{2} - 3\frac{3}{4}$ or $4\frac{3}{4}$ miles an hour; hence $1\frac{1}{4} \div 4\frac{3}{4} = \frac{5}{19}$ of an hour = $15\frac{15}{19}$ minutes. *Ans.*

6.

£0 17*s.* 10½*d.*
3
2 13 7½
6
16 1 9
3 qts. | ¼ | 0 13 4⅞
1½ pt. | ¼ | 0 3 4 7/32
£16 18*s.* 6 3/32*d.* *Ans.*

7. The whole sum $= (1+\frac{4}{9}+\frac{3}{5}$ of $\frac{4}{9})$ of the first share,
$= 1\frac{32}{45}$ of the first; hence $121s.\ 11d.+\frac{77}{45}$
$= 11s.\ 1d. \times \frac{45}{7}$

$= 1s.\ 7d. \times 45 = £3\ 11s.\ 3d.$, the 1st,
$1s.\ 7d. \times 20 = 1\ 11\ 8$ the 2nd,
$1s.\ 7d. \times 12 = 0\ 19\ 0$ the 3rd. } *Ans.*

8. $£5\ 11s.\ 5d. : £1\ 3s.\ 10\frac{1}{2}d.$
$6\frac{3}{4}$ dwt. : $1\frac{3}{4} \times 20$ dwt. } :: 18 car. : 20 car. *Ans.*

9. By the Chain Rule:

? lb. = 33 app.
if 54 app. = 78*d.*
88*d.* = 60 app.
13 app. = 4 lbs.

$\frac{33 \times 78 \times 60 \times 4}{54 \times 88 \times 13} = 10$ lbs. *Ans.*

10. $298\frac{3}{4}+956 = 1195 \div (956 \times 4) = \frac{5}{16} = .3125$. *Ans.*

11. $25{\cdot}18 \times 246 = 6194{\cdot}28$
$10s. = \frac{1}{2} = 12{\cdot}59$
$3s.\ 4d. = \frac{1}{3} = 4{\cdot}197$
$6d. = \frac{1}{20} = {\cdot}629$
$6211{\cdot}696$
or 6211 frs. 70 cts. *Ans.*

12. $8\frac{3}{4} \times .072+4\frac{1}{2}$
$= 35 \times .072+18$
$= 35 \times .004$
$= £.14 = 1$ flor. 4 cts. *Ans.*

13. $£1 \div (\frac{5}{12} \times \frac{4}{100}) = £1 \times \frac{1200}{20} = £60$. *Ans.*

14. $£195{\cdot}3125 \times .04$
$7{\cdot}8125$
$203{\cdot}125 \times .04$
$8{\cdot}125$
$211{\cdot}250 \times .04$
$8{\cdot}450$
$219{\cdot}700$

$£219\ 14s.\ 0d.$
$195\ 6\ 3$
$24\ 7\ 9$ = comp. int.
$23\ 8\ 9$ = simp. int.
$£0\ 19s.\ 0d.$ *Ans.*

15. 1 lb. tr. $= \frac{5760}{7000}$ of a lb. av.; hence $1-\frac{576}{700}$
$= \frac{31}{175}$ lb. av. $= \frac{496}{175}$ oz.; ∴ a lb. of feathers is heavier than a lb. of gold by $2\frac{146}{175}$ oz. av. *Ans.*

Again: 1 oz. tr. $= \frac{480 \times 16}{7000} = 1\frac{17}{175}$ oz. av.; ∴ an oz. of gold is heavier than an oz. of feathers by $\frac{17}{175}$ of an oz. av. *Ans.*

Thus, it appears that a lb. of feathers is about $21\frac{1}{2}$ per cent. heavier than a lb. of gold; but that an oz. of gold is nearly 10 per cent. heavier than an oz. of feathers.

16. The whole were bought at an average of $\frac{1}{2}(\frac{1}{4}+\frac{1}{3})$ of a penny per apple $= \frac{7}{24}d.$, and were sold at $\frac{1}{2}d.$; or the buying price was to the selling price as 49 : 48;

∴ 49 : 49−48 :: 100 : $2\frac{2}{7}$ per cent. loss. *Ans.*

17. The 4 per cent. stock is sold out for £42·75 × 80; and for every £99 which that cash contains, an income of £$5\frac{1}{2}$ is obtained; hence,

$$\frac{42{\cdot}75 \times 80 \times 5\frac{1}{2}}{99} = \frac{171 \times 10 \times 11}{99} = £190, \text{ 2nd income}$$

$42{\cdot}75 \times 4 = 171$, 1st income

Ans. £ 19, increase.

18. $100-7\frac{1}{2} : 100+12\frac{1}{2} :: 74s. : 90s.$ *Ans.*

19. The whole can be done by 1 man in 4 hrs., or by 1 woman in 6 hrs., or by 1 child in 12 hrs. Now, let the whole work be divided into 12 equal measures; 1 man does 3 of these per hour, 1 woman does 2, and 1 child does 1; ∴ 1 man 1 woman and 1 child can do 6 measures per hour, or the whole 12 measures in 2 hrs. *Ans.*

20. Let the whole work be divided into 90 equal measures. A can do 15 measures per hour, B 18, C 20. B in $2\frac{1}{2}$ hrs. could do 45, and C in $1\frac{1}{2}$ hr. could do 30. Hence, if B and C had continued *to the end of A's actual time*, 90+75 = 165 measures would have been done;
∴ 165+(15+18+20) = 5 hrs. *Ans.*

Society of Arts.

THE METRIC SYSTEM.

1. to 4. *See* Text-Book, pp. 276–8.

5. 1 cub. mètre = 1,000,000 cub. centimèt.

6. A litre = 1 cub. decimèt. = 61·027 cub. inches; hence $24 \times 24 \times 12 \div 61{\cdot}027 = 113{\cdot}26$ lit. *Ans.*

7. France, Holland, Belgium, Spain, Portugal, Italy, Greece, Germany, the United States, British India, &c. The Metric System would save a very great amount of time and labour in arithmetical instruction, in the application of calculation to science and commerce, &c.—*See* Professor Leone Levi's *Theory and Practice of the Metric System.*

8. Volume of air in the room = $(39{\cdot}3708)^3 \times 100 \times 4$, or 400 cubic mètres = 400000 cub. decimèt. = 400000 litres; and as a litre of air weighs about 1·293 grains, the whole weight = 517200 grains. *Ans.*

Note.—The weight 1·293 grains per litre = ·08071 lb. per cubic ft. *See* the Table of Specific Gravities in Text-Book, p. 279.

9. $20 \times 5 \times {\cdot}8 = 80$ cub. decimèt.; ∴ the quantity is 80 litres, and the weight 80 kilograms. *Ans.*

10. $3^3 = 27$ cub. decimèt. = $61{\cdot}027 \times 27$ cub. inches;

$$\therefore \text{ weight in lbs.} = \frac{61{\cdot}027 \times 27 \times 62\frac{1}{2}}{1728}$$

$$= \frac{61027 \times 27}{1728 \times 16} = \frac{61027}{64 \times 16} = 59\tfrac{1}{2} \text{ lbs. } \textit{Ans.}$$

11. $100^3 \times 3{\cdot}1416 \times 150$ cub. decimèt. = 4712400 litres. *Ans.*

12. The *gram* is the weight of a cubic centimètre of distilled water at freezing point. The *litre* is a cubic decimètre. The *are* is a square decimètre. *See* Text-Book, foot of p. 276.

Society of Arts.

MENSURATION.

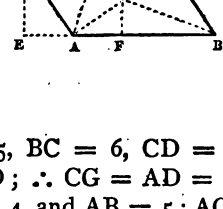
(Page 264.)

1. $17\frac{1}{4}$ ft. $\times$ $14\frac{7}{12}$ ft. $= (4025 \div 16)$ sq. ft. $= 251\frac{9}{16}$ sq. ft. $= 27$ sq. yds. 8 ft. 81 in. *Ans.*

Again: $251\frac{9}{16}$ @ 2*s.* 6*d.* $= 7546\frac{7}{8}d. = £31\ 8s.\ 10\frac{7}{8}d.$ *Ans.*

2. If AB $= 28$, BC $= 45$, CA $= 53$, the difference of the segments of the base made by a perpendicular from A on BC $= (CA+AB)(CA-AB) \div BC = 81 \times 25 \div 45 = 45 =$ BC; $\therefore$ ABC is a right angle, and the area $= \frac{1}{2}(45 \times 28) = 630$. *Ans.*

3. AB is the base of the quadrilateral ABCD, the sides AD and BC parallel.



By Eucl. I. 37, the triangles ADC and ADB are equal. Now, ADB $=$ AB $\times \frac{1}{2}$DE, and ACB $=$ AB $\times \frac{1}{2}$CF;

$\therefore$ ABCD $=$ AB $\times \frac{1}{2}$(DE+CF).

4. *See* preceding figure: AB $= 5$, BC $= 6$, CD $= 4$, AD $= 3$. Draw AG parallel to CD; $\therefore$ CG $=$ AD $= 3$; also GB $= 3$; and as AG $=$ CD $= 4$, and AB $= 5$; AGB is a right angle; $\therefore$ area of ABCD $= \frac{1}{2}$(AD+BC)AG $= 9 \times 2 = 18$. *Ans.*

5. $\left(94\frac{1}{2} \times 12\frac{3}{4} \times \frac{2\frac{1}{2}}{1\frac{1}{2}}\right) \div 272\frac{1}{4} = \frac{189}{2} \times \frac{51}{4} \times \frac{5}{3} \times \frac{4}{1089}$

$= 7\frac{91}{242}$ sq. rods. *Ans.*

6. The circuit $= (5\frac{1}{2}+3\frac{1}{3})\times 2 = 17\frac{2}{3}$ ft.
Area of sides and ends $= 17\frac{2}{3}\times 1\frac{1}{8} = 20\frac{11}{18}$ sq. ft.
Area of bottom $= 5\frac{1}{2}\times 3\frac{1}{3} = 18\frac{1}{3}$
Lead required, $38\frac{17}{18}$ sq. ft.

$$\text{Cost} = £2\times\frac{38\frac{17}{18}\times 12}{112} = \frac{£701}{84} = £8\ 6s.\ 10\frac{8}{7}d.\ \textit{Ans.}$$

7. Radius of greater circle $= \sqrt{(154\div 3{\cdot}1416)}$
$= \sqrt{(1\div{\cdot}0204)} = \frac{1}{2}\sqrt{(1\div{\cdot}0051)}$;
radius of inner circle $= \frac{1}{3}\sqrt{(616\div 3{\cdot}1416)} = \frac{1}{3}\sqrt{(1\div{\cdot}0051)}$.
Difference $= \frac{1}{6}\sqrt{(1\div{\cdot}0051)} = \frac{1}{6}$ of ·14 nearly, $= 2\frac{1}{3}$ yds. or 2 yds. 1 ft. nearly. *Ans.*

8. The diameter of the inner semicircle is 2 inches, and the height of the segment of it which forms the hollow part is $\frac{1}{2}$ an inch: to find the area of that segment.

Here we have the chord subtending an arc of 120°; so that the sector is $\frac{1}{3}$ of the circle
$= \frac{1}{3}$ of $3{\cdot}141592 = 1{\cdot}047197$ sq. in.; and the area of the triangle by which the sector exceeds the segment is $\frac{1}{4}\sqrt{3}$ $= {\cdot}433013$ of a sq. in.; ∴ area of segment $= {\cdot}614184$ of a sq. in. *Ans.* [Or, *Ans.* $= \frac{1}{12}(4\pi-3\sqrt{3})$].

9. Volume $= \sqrt{(3\frac{8}{9}\times 18\frac{1}{3}\times 6\frac{5}{12})} = \frac{1}{18}\sqrt{(35\times 55\times 77)}$
$= \frac{1}{18}\sqrt{5^2\times 77^2} = \frac{1}{18}$ of $5\times 77 = 21\frac{7}{18}$ cu. ft.
$= 21$ cub. ft. 672 in. *Ans.*

10. $21\frac{7}{18}$ cu. ft. divided severally by $3\frac{8}{9}$, $6\frac{5}{12}$, and $18\frac{1}{3}$ sq. ft. gives $5\frac{1}{2}$ ft., $3\frac{1}{3}$ ft., and $1\frac{1}{6}$ ft., the dimensions of the block. Therefore, the height of the least cylinder would be $1\frac{1}{6}$ ft. *Ans.* Also, its diameter would be $\sqrt{(\frac{11}{3})^2+(\frac{10}{3})^2}$ $= \sqrt{41{\cdot}361111} = 6{\cdot}43126$ ft. *Ans.*

11. The regular tetrahedron being a pyramid whose base and sides are four equilateral and equal triangles, we have $\sqrt{10^2-5^2} = \sqrt{15\times 5} = 5\sqrt{3}$ the perpendicular altitude

of each triangular face; $\therefore$ the height of the pyramid $= \sqrt{(75 - \frac{25}{4} \times 3)} = 7\frac{1}{2}$ ft.

The area of the base $= 5 \times 5\sqrt{3}$; and this $\times$ a third of the height $= 5 \times 5\sqrt{3} \times 2\frac{1}{2} = 31\frac{25}{2}\sqrt{3}$, the solidity in cu. ft. $= 108{\cdot}2532$ cu. ft. $= 108$ cu. ft. $437\frac{1}{2}$ in. *Ans.*

Also, the surface $= 5 \times 5\sqrt{3} \times 4 = 100\sqrt{3}$ in sq. ft. $= 173{\cdot}2051$ sq. ft. $= 173$ sq. ft. $29\frac{1}{2}$ in. *Ans.*

12. The vessel being a frustum of a cone, let h = the height; then

$(6^2 + 7^2 + 6 \times 7) \times {\cdot}785398 \times \frac{1}{3}h = \frac{1}{8}$ of $277{\cdot}274$;

$$\therefore h = \frac{34{\cdot}659 \times 3}{127 \times {\cdot}785398} = \frac{103{\cdot}977}{99{\cdot}745} = 1{\cdot}0424 \text{ in. } \textit{Ans.}$$

13. Solidity of the plate in cu. in. $= 6^2 \times \pi$; and if D denote the diameter of the sphere, its solidity will be $D^3 \times \frac{1}{6}\pi$; $\therefore \frac{1}{6}D^3 = 6^2$, or $D = 6$ in. *Ans.*

14. *See* the Text-Book, p. 244.

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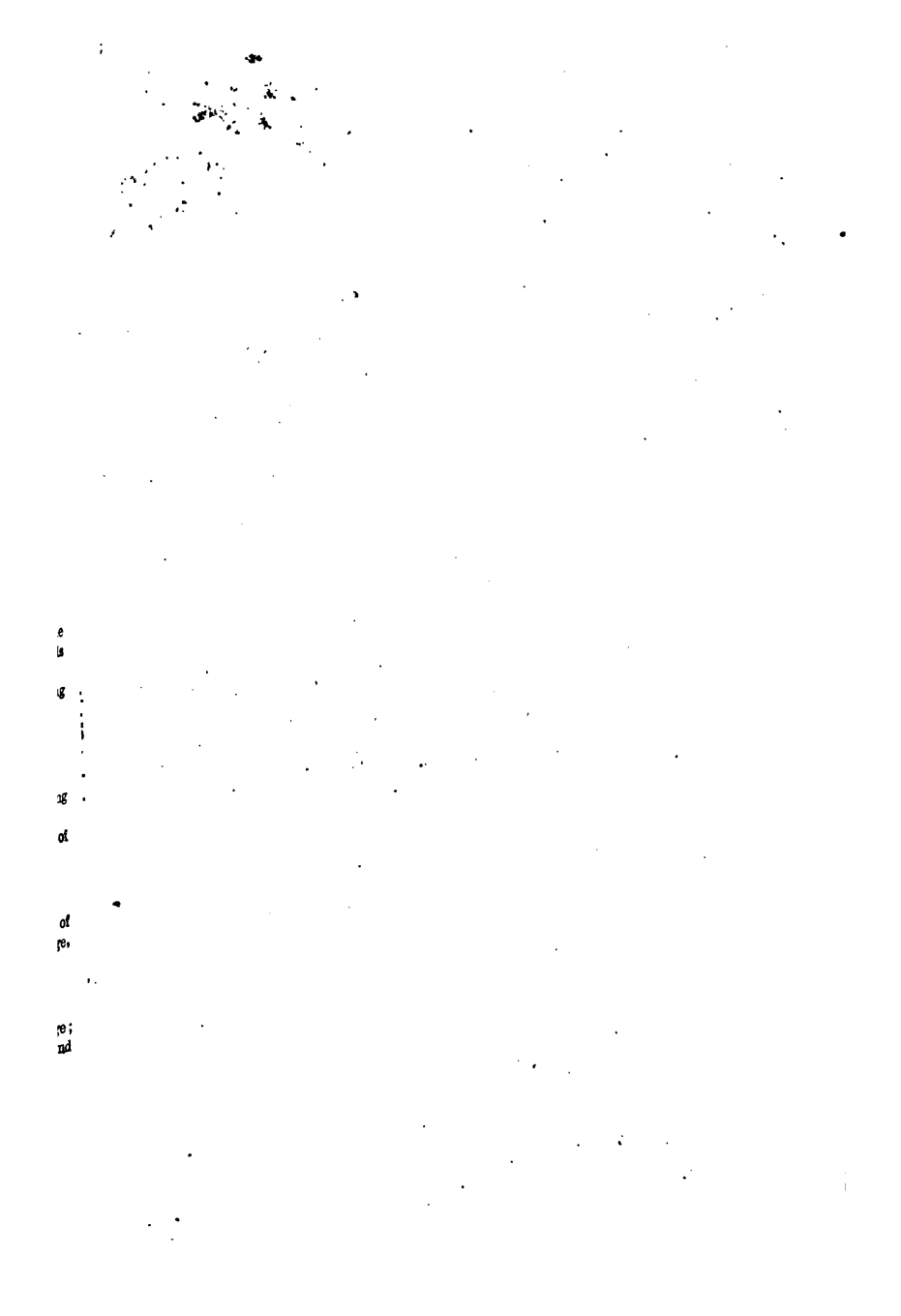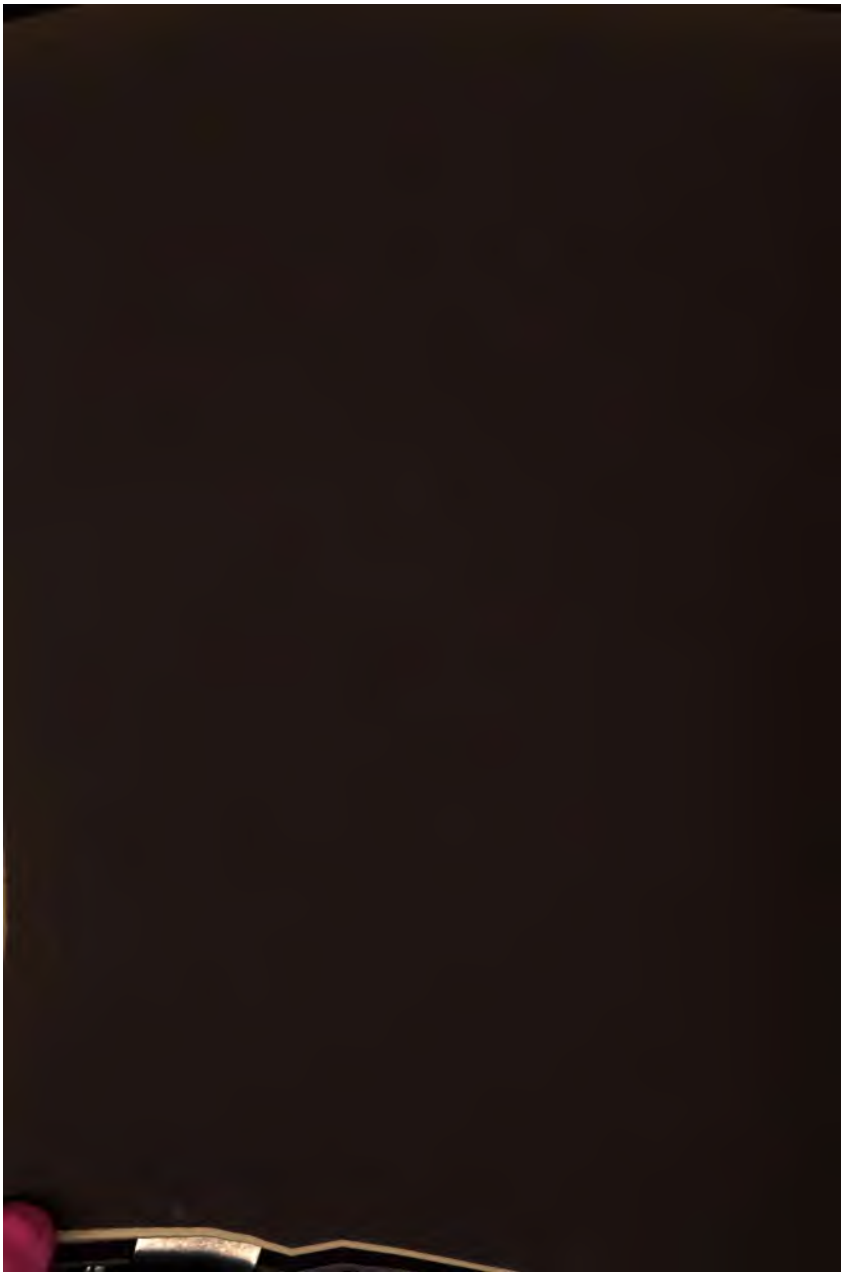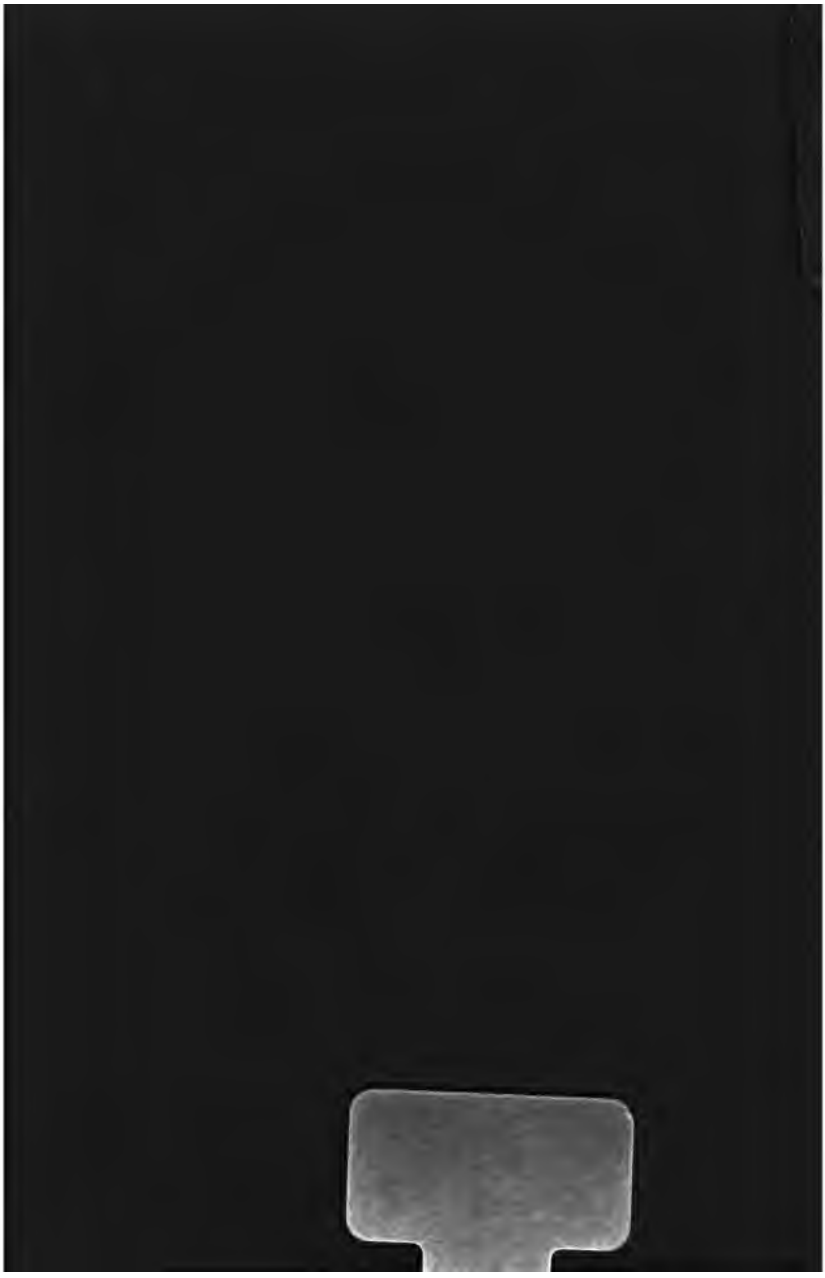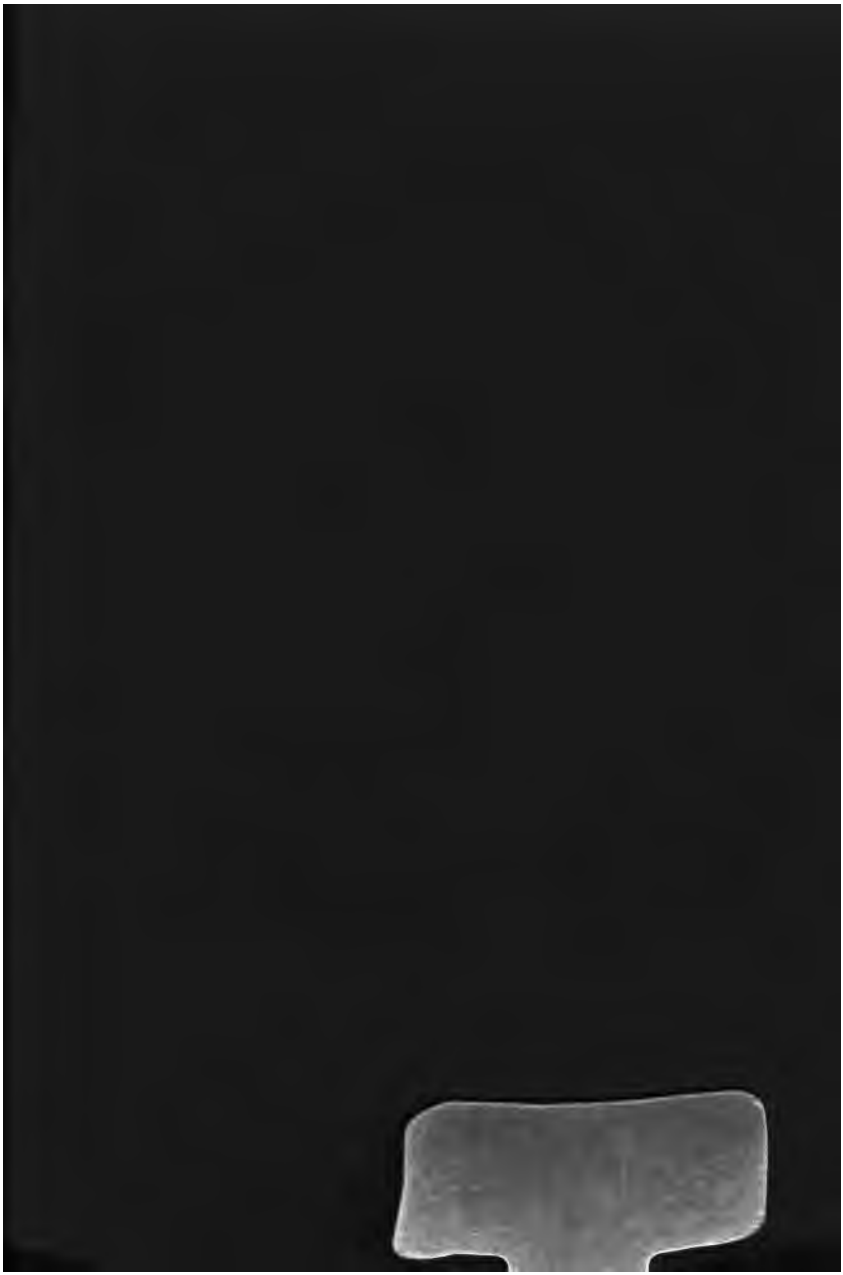
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